Exam III and Key April 3, 2006
MAC 2312 S Hudson

1) (5pt each) Compute (or state that it diverges):

a) $\int \frac{x^2+1}{x-1}\ dx$

b) $\int_0^{+\infty} e^{2x}\ dx$

c) $\int_0^1 \frac{dx}{\sqrt{1-x^2}}$

d) $\int \frac{x^2}{(x+3)^3}\ dx$

2) (5pt each) Compute (or state that it diverges):

a) $\sum_{k=1}^{\infty}(-3/4)^{k-1}$

b) $\sum_{k=1}^{\infty} \frac{1}{(k+2)(k+3)}$

3) (10pt) Find the general term a_k of this sequence, and its limit L.

$$1,\ -1/2,\ 1/3,\ -1/4,\ 1/5\ldots$$

4) (10pt) Show that $\{2n^2 - 7n\}$ is eventually monotone.

5) (10pt) Compute $\int \frac{dx}{x^2-4x+5}$

6) (5pt each) State whether the series converges or diverges, with a brief reason (eg a theorem or a test):

a) $\sum_{k=3}^{\infty} \frac{1}{\sqrt{k}}$

b) $\sum_{k=1}^{\infty} \frac{k^2}{2k^2+1}$

c) $\sum_{k=3}^{\infty} \frac{(-1)^k}{2k+1}$

d) $\sum_{k=1}^{\infty} \frac{1}{k\ \ln(k)}$

7) (10pts) Use the Trapezoid Rule to approximate $\int_1^2 x^2\ dx$ with $n = 2$.

Based on your knowledge of $y = x^2$ (increasing, concave up, etc), and perhaps a picture, explain whether this approximation is too large or too small.

8) (10pts) Choose ONE, and explain thoroughly:

a) State and prove thm 10.3.3 about geometric series.

b) State and prove the Divergence Test.

c) Prove that the Harmonic Series diverges.

Remarks and Answers: The average was approx 63/100.

1a) Divide before integrating, $x^2/2 + x + 2\ln|x-1| + C$. It's also OK to start with $u = x - 1$, which leads to an equivalent answer (that looks a bit different). Note: *Never* write "Diverges" for an indefinite integral like this one.

1b) Start with $\lim_{M\to+\infty} \int_0^M e^{2x}\, dx$ etc. It diverges.

1c) This is also improper (not defined at $x = 1$) and requires a limit for full credit, but it ends with $\sin^{-1}(1) - \sin^{-1}(0) = \pi/2$

1d) Use P.F's with $A = 1$, $B = -6$ and $C = 9$. Get $\ln|x+3| + 6(x+3)^{-1} - (9/2)(x+3)^{-2} + C$.

2a) It's geometric: $S = a/(1-r) = 1/(1+(3/4)) = 4/7$. Since this is alternating, you can check your answer from the theorem that S lies between the partial sums, $1 = s_1 > S > s_2 = 1/4$.

2b) Use P.F's to split it into $1/(k+2) - 1/(k+3)$. Then it telescopes and $S = 1/3$.

3) $\lim(-1)^{k+1}/k = 0$

4) Of course, you have to provide a clear reason here - not just a few terms and then "so it is monotone". There are several possible strategies, explained in Ch.10.2. For example, $a_{n+1} - a_n = 2(n+1)^2 - 7(n+1) - 2n^2 + 7n = 4n - 5 > 0$ when $n \geq 2$. This shows that a_n increases after

a_2. It is *not* OK to use $\lim a_n = \infty$ as an explanation [this is also true for $a_n = n/100 + (-1)^n$, for example, which is not monotone].

5) Complete the square, $(x-2)^2+1$, and do a u-sub (or trig-sub) to get $\tan^{-1}(x-2)+C$. It seems reasonable to try factoring (and then use P.F's), but it doesn't factor.

6) DDCD, with these reasons : p-series (or Int.T), Div.T, AST, Int.T. [include the calcs].

Note on 6d): You cannot compare this with $1/k$ because $1/k$ is *larger*. Nor does the LCT help here.

7) 19/8; It is too large, since f is concave up. It is easy to check this, since the exact area is $\int_1^2 x^2 = 7/3$. Checking is not required, but it is a good habit, and it may affect your partial credit in such problems.

8) See text.