Exam III — Aug 2, 2011
MAC 2312 — S Hudson

Remember that some integrals may be improper, and that some could benefit from a u substitution (or some algebra) before applying fancier methods. When in doubt, $u = e^x$ or $u = x^2$ is often a good try.

1) (10pts each) Compute the integrals:

a) $\int x^3 e^{x^2} dx$

b) $\int \tan^5\theta \sec\theta \; d\theta$

c) $\int \frac{e^{3x}\, dx}{e^{2x}+1}$

2) (5pts each) Compute the integral (or show that it diverges):

a) $\int_{-1}^{+1} \frac{dx}{x^2} =$

b) $\int_0^{+\infty} \frac{e^{-\sqrt{x}} dx}{\sqrt{x}} =$

3) (10 pts) Start on this by showing the partial fraction splitting, with A, B etc. But you don't have to compute these constants or get an antiderivative; $\int \frac{2x-3}{x^3-x^2} =$

4) [10 pts] Approximate $\int_{-1}^{+1} e^{-x^2}\, dx$ by S_4 (Simpson's Rule with $n = 4$). You can use any of the following data from my calculator that you need. Don't leave any variables in your answer, but a few plus signs and fractions are OK.

$e^1 = 2.7$	$e^{1/2} = 1.6$	$e^{1/4} = 1.3$	$e^{1/16} = 1.06$
$e^{-1} = 0.37$	$e^{-1/2} = 0.61$	$e^{-1/4} = 0.78$	$e^{-1/16} = 0.94$

5) [5 pts each] a) Decide if this converges or not and explain. If it converges, find the sum; $\sum_{k=1}^{\infty} (2/3)^{k+2}$

5b) What does the Divergence Test tell you about $\sum_{k=1}^{\infty} \frac{k}{e^k}$? Answer with C or D or "inconclusive", and explain. You do not have to prove every step (eg, some easy calculations might be done by guesswork).

6) (10pts) Choose ONE proof, explain thoroughly. As usual, you may use the back, but leave a note.

a) State and prove the Comparison Test.

b) State and prove the formula for the sum of an infinite geometric series for the case $|r| < 1$.

7) [20pts] Answer True or False:

$\int_0^{\infty} x^{-1/2}\, dx$ is improper, but converges.

$\int_{-\infty}^{\infty} \frac{dx}{1+x^2}$ diverges.

If $\sum a_k$ converges and $a_k \geq 0$, then $a_k \to 0$.

If $0 < k^2 a_k < 1$ for all k, then $\sum a_k$ converges.

There is a convergent series $\sum a_k$ with unbounded partial sums.

There is a bounded sequence that diverges.

A constant sequence is both increasing and decreasing, but not strictly decreasing.

The sequence $a_k = |k - 3|$ for $k \geq 1$ is not monotone, but is eventually increasing.

The series $1 - 1 + 1 - 1 + 1 - 1 + \ldots$ converges to 0.

Every monotone sequence that is bounded above converges.

BONUS: (5 pts) I was playing around with the Bishop's example [done in class] and found this similar series. What is its sum ?

$$1 + 2/3 + 3/9 + 4/27 + \ \ldots = ?$$

Remarks and Answers: The average was about 64 out of 100. The grades were a bit low on problems 1c and 2. The unofficial scale is:

A's: 77-100
B's: 67-76
C's: 57-66
D's: 47-56

I added your 3 exam scores together to estimate your semester grade (I will include HW later, of course). Your total Σ is noted in the upper right corner of page 1, with a letter based on this scale;

A's: 240-300
B's: 210-239
C's: 180-209
D's: 150-179

You should probably check that I have your total correct, and that the letter matches this scale.

1a) $e^{x^2}(x^2 - 1)/2 + C$. Start with a u-sub (see hint), $u = x^2$, to get $\int ue^u$. Next, use IBP.

Many people started with IBP, and got stuck (or went on badly) and didn't get many points. If an idea just doesn't work, try another! A few people found a clever way to start with IBP ($2xe^{x^2} \times x^2/2$) and got full credit.

1b) Set $u = \sec\theta$, get $\int (u^2-1)^2\ du = u^5/5 - 2u^3/3 + u + C$, and replace u.

1c) $e^x - \tan^{-1}(e^x) + C$. Set $u = e^x$ (see hint) and get $\int \frac{u^2\ du}{u^2+1} = \int 1 - \frac{1}{u^2+1}\ du$, which is now easy.

2a) Diverges. Since f is unbounded near 0, this is improper. Split it into $\int_{-1}^{0} x^{-2}\ dx + \int_0^1 x^{-2}\ dx = A + B$. Compute $B = \lim \int_\epsilon^1 x^{-2}\ dx$ and show this diverges (or do this for A instead).

2b) Set $u = \sqrt{x}$. Use a limit to get $2\int_0^\infty e^{-u}\ du = 2$ (improper again).

3)

$$\frac{A}{x} + \frac{B}{x^2} + \frac{C}{x-1}$$

4) (2/12)(0.37 + 2(0.78)+ 4 + 2(0.78) + 0.37)

5a) 8/9, 5b) Inconclusive, because $\lim \frac{k}{e^k} = 0$.

6) see text

7) FFTTF TTTFF

Bonus) $(3/2)^2$