

Name _____

Show all your work and reasoning for maximum credit. Especially, explain any proofs. Do not use a calculator, book, or any personal paper. You may ask about any ambiguous questions or for extra paper.

1) (20pts) Compute and simplify. Use limits on any improper integrals.

$$\int_{-\infty}^{+\infty} \frac{1}{1+x^2} dx$$

$$\int_0^{\ln(2)} e^{2x} dx$$

$$\frac{d}{dx} \int_0^{x^2} e^{t^2} dt$$

$$\int_0^{\pi/4} \tan^2(x) dx$$

$$\int_0^2 |x^2 - 1| dx$$

2a) (10pts) Find the McLaurin Series (eg the Taylor Series at 0) for $f(x) = e^{-x^2}$.

2b) Use your answer to 2a) to approximate $\int_0^1 e^{-x^2} dx$ to two decimal places. You do NOT have to simplify the sum, but if your answer has 7 summands (for example) explain your reasoning for including those 7. I think you do not need a calculator (eg $4! = 24$ and $5! = 120$ etc are pretty easy) but you can ask me for some numerical help.

3) (10pts) Find the area of the region that is inside one loop of the rose $r = \cos(3\theta)$.

4) (10pts) A cone-shaped reservoir is 20 ft in diameter across the top and 15 feet deep. If the reservoir is filled to a depth 10 ft, how much work is required to pump all the water to the top of the reservoir? (Assume density of water is 62.4 lb per cubic ft).

5) (10pts) Find the second Taylor polynomial ($p_2(x)$) for $\tan(x)$ centered at $\pi/4$. Suggestion: Compute c_k directly. Hint: $\sec(\pi/4) = \sqrt{2}$.

6) (10pts) Suppose a particle has position $s(t)$ after t seconds. Suppose the acceleration is $a(t) = 2t$ and the initial velocity is $v_0 = 3$ and the initial position is $s(0) = s_0 = 4$. Find a formula for $s(t)$ and then find the displacement for the period $0 \leq t \leq 2$.

7) (10pts) Answer True or False.

The harmonic series converges conditionally.

The average value of $\cos(x)$ on $[0, 2\pi]$ is zero.

If $|r| < 1$ then $r^2 + r^3 + r^4 + \dots$ converges to $\frac{r^2}{1-r}$.

$$\lim_{x \rightarrow \infty} \left(1 + \frac{1}{x}\right)^{2x} = e^2$$

The polar equations $r = \cos 2\theta$ and $r = -\cos 2\theta$ have the same graphs.

8) (10pts) Choose ONE to do;

- A. State and prove Rolle's Thm
- B. State and prove the Comparison Test
- C. State and prove the formula for arc length.

9) (10pts) Find the interval of convergence for the series $\sum \frac{(-1)^{k+1}(x+1)^k}{k}$.