

1) (15pts) Compute and simplify. Use limits on any improper integrals.

$$\int_{-\infty}^{+\infty} \frac{1}{1+x^2} dx$$

$$\int_0^{\ln(2)} e^{2x} dx$$

$$\int \frac{2}{x^2-1} dx$$

2) (short answer; 5pts each)

2a) Find the McLaurin Series for $f(x) = x \sin(2x)$.

2b) Use a derivative of a familiar series to get the McLaurin Series for $f(x) = (1-x)^{-2}$.

2c) Give the McLaurin series for e^x . Use a partial sum $p_n(x)$ to estimate $1/e = e^{-1}$ to two decimal places. Choose n as small as you can and explain why your answer is accurate enough. Hints; $5! = 120$ and $7! = 5040$.

3) (10pts) Find the area of the region inside the cardioid $r = 1 + \cos(\theta)$.

4) (10pts) Choose ONE:

4a) A cone-shaped reservoir is 20 ft in diameter across the top and 15 feet deep. If the reservoir is filled to a depth 10 ft, how much work is required to pump all the water to the top of the reservoir? (Assume density of water is 62.4 lb per cubic ft).

4b) A 20 ft chain weighs 10 lbs per foot, so 200 lbs. One end is at the top of a tall building, and the other end hangs in the air 20 ft below that. How much work is required to lift the entire chain to the top ?

5) (10pts) Find the slope of the tangent line to the polar curve $r = 2 \cos(\theta)$ at $\theta = \pi/3$. For full credit, use chain rule formula for dy/dx , as in class.

6) (20pts) Answer with D, CA or CC, and justify. All sums go from $k = 1$ to ∞ :

$$\sum \frac{(-1)^k}{2k+1}$$

$$\sum \left(\frac{-2k}{3k+1}\right)^k$$

$$\sum \frac{2k}{3k+1}$$

$$\sum \frac{\sin(k^2)}{k^2}$$

7) (10pts) Use R_n to show the McLaurin series for $\sin(x)$ converges to $\sin(x)$ for all x .

8) (10pts) Choose ONE to do;

A. State and prove the integral formula for area in polar coordinates.

B. State and prove the Comparison Test.

Remarks and Answers: The average was about 62 / 100, which is lower than the other Summer B exams, but fairly normal for a final. I do not usually compute a separate scale for the final. Also, I cannot set a semester scale until I get the HW totals for the semester, but that should happen with 1-2 more days.

1a) π ; improper, use \lim and \tan^{-1} .

1b) $3/2$; use $u = 2x$ and $e^{2\ln 2} = 4$

1c) $\ln|x-1| - \ln|x+1| + C$; use PF's.

2a) $2x^2 - 8x^4/3! + 32x^6/5! \dots$, by subbing $2x$ into \sin and then raising each power by one. I took off a point for answers like $x(2x - (2x)^3/6 + \dots)$, which is not in standard form. A prof might also deduct points for omitting the ellipsis (the dots), but I didn't do so this time.

2b) $1 + 2x + 3x^2 + \dots$, from the series for $1/(1-x)$, followed by a derivative of both sides. You could square both sides instead, but this is harder to do, and violates the instructions. The answer is NOT simply $1 + x^2 + x^4 \dots$

2c) Using the R_n formula, with $M = 1$ and $1/(n+1)! \leq 0.005 = 1/200$, I get $n = 5$ (but I accepted some other similar answers). So, we can use $p_5(-1) = 1 - 1 + 1/2 - 1/6 + 1/24 - 1/120$. Simplification not required. It is also OK [and faster] to use the AST error estimate instead of R_n (with the same conclusion).

3) $\int_0^{2\pi} (1 + \cos \theta)^2/2 \, d\theta = 3\pi/2$

4a) $W = \int_0^{10} 62.4\pi(2y/3)^2(15 - y) dy$ (etc)

4b) $W = \int_0^{20} 10y dy = 2000$ ft-lbs.

5) $1/\sqrt{3}$; The P.Eqns are $x = 2 \cos^2 \theta$ and $y = 2 \cos \theta \sin \theta$. Then

$$\frac{dy}{dx} = \frac{dy/d\theta}{dx/d\theta} = \frac{\cos^2 \theta - \sin^2 \theta}{-2 \cos \theta \sin \theta} = \frac{1}{\sqrt{3}}$$

It is also OK to memorize and use the formula from 10.3, for a slight shortcut.

6) CC, CA, D, CA. Here are some fairly-detailed comments on part a).

6a) This converges, by the AST (since $1/(2k + 1)$ decreases to 0). The series $\sum 1/(2k + 1)$ looks like the Harmonic Series, which diverges. We check this by the LCT; $\lim_{k \rightarrow \infty} \frac{1/(2k+1)}{1/k} = 1/2$. So, the series does not CA; it CC's.

Each of the sub-problems, such as 6a, was 5 points. I gave 3/3 for writing CC (but 0/5 without that). I gave another 2/2 if you justified CC. Name the test[s] you use, and provide enough work. I gave approx 1/2 for 'this is like the AHS', which is vague, but true. The others were graded similarly; for full credit on the fourth, mention p-series AND CT.

7) See the text for a model proof. I graded mainly on these two features;

- a) $R_n(x) \leq M|x|^{n+1}/(n + 1)!$, and
- b) $\lim |x|^{n+1}/(n + 1)! = 0$

and also, a few points for your words, a note about $M = 1$, etc.