Exam I | Jan 30, 2003
MAC 2312 | S Hudson

Name

Show all your work. Use the space provided, or leave a note. Don't use a calculator or your own extra paper.

1) Short calculations, 5 pts each. Simplify any easy terms like $\sin(\pi)$ and e^0.

a) $\frac{d}{dx}\ [\ \int_0^x \sec^2(t)\ dt\ ]$

b) $\int_0^4 \sqrt{x} + e^x\ dx$

c) $\int_1^2 \frac{1}{t}\ dt$

d) $\int_0^{\pi/2} \sin^2 x\ \cos x\ dx$

e) $\int_0^3 |x-2|\ dx =$

f) Find the average value of $f(x) = x^3$ on the interval [0,5].

g) $\int_0^4 \frac{dx}{\sqrt{2x+1}} =$

h) $\int_0^\pi \sin^2 3x\ dx$

i) Find the area between the parabolas, $y = 2x - x^2$ and $y = 2x^2 - 4x$.

j) Use left-hand endpoints to find $\underline{A}_{\ 2}$ for $f(x) = \sin(x)$ on $[0, \pi/2]$.

2) (10pts) Rosanne drops a ball from a height of 400 ft. The force of gravity is 32ft/s/s, which is the acceleration on the ball. So, the velocity is $-32t$ (or, you might use $32t$).

a) Find a formula for the height of the ball $h(t)$ after t seconds. Be sure to show all your work and reasoning, and check that $h(0) = 400$.

b) How many seconds until the ball hits the ground? (This occurs when h=0, of course).

3) (10pts) a) Let $f(x) = x^2$ on [0,2]. Use the well-known formula $n(n+1)(2n+1)/6$ to calculate and simplify the Riemann Sum below. Answer in terms of n

$$\sum_{i=1}^{n} f(x_i)\Delta x$$

b) Find the exact area under this curve by taking a limit as $n \to \infty$. [Do NOT use the FTC, except maybe to check your answer].

4) (15 pts) Answer True or False in the margin. You don't have to explain. You can set $n = 10$ in the last three if you like.

There is a point c in [0,1] such that $\sin(c) = \int_0^1 \sin(x)\ dx$.

$f(x) = 1/x$ is integrable on the interval [-1,1].

If f is linear, then Simpson's rule will produce an exact answer for $\int_a^b f(x)\ dx$.

The midpoint rule produces an exact answer for $\int_0^2 3x^2\ dx$.

The midpoint rule produces a smaller estimate for $\int_1^4 5 - x^2\ dx$ than the Trapezoid rule.

5) (15pts) Choose ONE proof. Explain thoroughly. You can continue on the back.

A) State and prove the FTC, Part One, about computing the derivative of a definite integral. [You can use other definitions and theorems in your proof - but point out when you are doing so]

B) State and prove the formula for $1 + 2 + \ldots + n$.