Exam II | Feb 24, 2003
MAC 2312 | S Hudson

Name

Show all your work. Use the space provided, or leave a note. Don't use a calculator or your own extra paper. You may use these formulae whenever needed:

$$\int u\sqrt{a+bu}\,du = \frac{2}{15b^2}(3bu-2a)(a+bu)^{3/2}+C, \quad \int \frac{u\,du}{\sqrt{a+bu}} = \frac{2}{3b^2}(bu-2a)\sqrt{a+bu}+C$$

1) (15pt) Find the volume, when the region bounded by these curves is revolved around the y axis: $y = x^3$, $x = 0$, $y = 1$.

2) (10pt) Find the arc length of the curve $y = \frac{x^3}{6} + \frac{1}{2x}$ from $x = 1$ to $x = 3$.

3) (10pt) A cone-shaped water reservoir is 20 ft in diameter at the top and 20 ft deep. If it is filled to a depth of 10 ft, how much work is required to pump all the water to the top? [water weighs 62 lbs/ft^3].

4) (5pts each) Compute the integral, and make any obvious simplifications:

a) $\int xe^{-x}\,dx$

b) $\int \sin^2(x)\,dx$

c) $\int x\sqrt{2x-3}\,dx$

5) (10pts) Find the volume V of the solid generated by revolving around the y-axis the region under $y = 3x^2 - x^3$, from $x = 0$ to $x = 3$.

6) (10pts) Given the error estimate below, find the maximum possible error in using Simpson's rule to approximate $\ln(2) = \int_1^2 1/t\,dt$ with $n = 10$ subintervals. Hint: $f^{(4)}(x) = 24x^{-5}$.

$$|ES_n| \leq \frac{M_4(b-a)^5}{180n^4}$$

7) (15pts) Choose ONE, and explain thoroughly:

a) Explain the integral formula used in the Shell Method. Include a picture, a limit, a sum, the volume of a shell, and plenty of words in your answer.

b) Prove the formula $\ln(ab) = \ln(a) + \ln(b)$ using the definition of ln

c) Use the formula for the derivative of f^{-1} and of $\ln(x)$ to prove the derivative of e^x is e^x.

8) (15pts) Answer True or False:

$\ln(xe^{3x}) = \ln(x) + \exp(\ln(3x))$ for all $x > 0$.

$\log_2 3 \times \log_3 4 = 2$

Simpson's rule with $n = 10$ produces an exact answer for $\int_2^5 5x^3 + 7 \, dx$.

$\int \ln(x) \, dx = x \ln(x) + x - C$.

The best approach to $\int \sin^3 x \cos^2 x \, dx$ is to set $u = \sin(x)$.

BONUS (5 pts; not supposed to be easy): Evaluate $\int \sqrt{1 + e^x} \, dx$ by finding the correct u-substitution. You might try; $u = e^x$, $u = 1 + e^x$ or $u = \sqrt{1 + e^x}$.