Exam II, 100 minutes, (Key) Feb 25, 2003

*The average was about 60/100, with mostly good scores except on problems 2 and 3.*

1) Draw a picture, use disks, $\int_0^1 \pi(y^{1/3})^2\, dy = 3\pi/5$. Or shells, $\int 2\pi x(1 - x^3)\, dx =$ same.

Common mistakes - setting it up wrong, in seemingly random ways (using dx with disks, using $x^3$ instead of $y^{1/3}$ etc.

2) This was HW problem Ch 6.4-23. I gave about 5/10 points if you got the first two steps right (which *should* have been easy).

$$\begin{aligned} L &= \int_1^3 [1 + (f')^2]^{1/2}\, dx \\ &= \int_1^3 [1 + (x^2/2 - x^{-2}/2)^2]^{1/2}\, dx \\ &= \int_1^3 [x^4/4 + 1/2 + x^{-4}/4]^{1/2}\, dx \\ &= \int_1^3 x^2/2 + x^{-2}/2\, dx \\ &= x^3/6 - x^{-1}/2 \,|_1^3 = 27/6 - 1/6 - 1/6 + 1/2 = 14/3 \end{aligned}$$

Simplification is suggested mainly for a "sanity check" of the answer. Mistakes - getting $f'$ wrong.

3) I gave about 6/10 points for the first line:

$$\begin{aligned} W = \int \text{weight dist dy} &= \int_0^{10} 62\pi(y/2)^2(20 - y)\, dy \\ &= 15.5\pi(20y^3/3 - y^4/4)|_0^{10} = etc \end{aligned}$$

4) Not much partial credit on these, unless the mistake was *very* minor, such as forgetting the +C, or a minus sign. If time permits, check your answers to such problems by taking a derivative.

4a) Use IBP: $xe^{-x} + \int e^{-x} = -xe^{-x} - e^{-x} + C$

4b) $\int (1 - \cos(2x))/2 \; dx = x/2 - \sin(2x)/4 + C$

4c) Use the formula on page one with $a = -3$ and $b = 2$ and $u = x$; get

$$\frac{1}{30}(6x + 6)(2x - 3)^{3/2} + C$$

5) [Example 1 of 6.3] Use shells. $V = \int_0^3 2\pi x(3x^2 - x^3) \; dx = 2\pi(3(3)^4/4 - 3^5/5) = 243\pi/10$. Common mistake: Using $\int_0^3 \pi(3x^2 - x^3)^2 \; dx$, which would be OK if it went around the $x$-axis.

6) [Example 7 of 5.9] Since $f^{(4)}$ is decreasing, its maximum is $M_4 = f^{(4)}(1) = 24$. So, 24/1,800,000.

7a) (Advertised) See lecture notes or pages 420-421.

7b) (Advertised) See lecture notes or page 458.

7c) [The textbook is not very rigorous about this.] See formula 14 on page 182, and set $f(x) = \ln(x)$ and $g(x) = e^x$. Get

$$g'(x) = \frac{1}{f'(g(x))} = \frac{1}{1/(e^x)} = e^x$$

8) TTTFF

Bonus) Set $u = \sqrt{1 + e^x}$, so $du = (1/2)(1 + e^x)^{-1/2} e^x \; dx = (u^2 - 1)/2u \; dx$. Then $\int \sqrt{1 + e^x} \; dx = \int 2u^2/(u^2 - 1) \; du$. (Four points for getting this far). Then, partial fractions: $2u + \ln[(u - 1)/(u + 1)] + C$, etc. In principal, the other substitutions should work out eventually, too, but they seem even harder.