Exam III — Apr 1, 2003

MAC 2312 — S Hudson

Name

Show all your work. Your method may count as much as your final answer. Use the space provided, or leave a note. Don't use a calculator or your own extra paper.

1) Find the arc length of the curve $y = \frac{x^3}{6} + \frac{1}{2x}$ from $x = 1$ to $x = 3$.

2) $\int \tan^5 t \sec^4 t \; dt$

3) $\int \frac{\sqrt{x^2-9}}{x}\ dx$

4) $\int \frac{5x-4}{x^2-4x}\ dx$

5) $\int \frac{2x+5}{x^2+4x+5}\ dx$

6) $\int_0^\infty xe^{-3x}\ dx$

7) Choose ONE, and explain thoroughly:

a) State and prove the Divergence Test.

b) Prove that $\lim_{n\to\infty} 1/n = 0$ (using the ϵ method).

c) State and prove the M-test (you may quote theorems about sequences).

8) CHOOSE ONE of these to do;

A) $\int e^x \ \sin x \ dx$

B) Find the reduction formula for $\int \tan^n x \ dx$ using I.B.P.

9) (20 pts total) Answer True or False:

A good first step in solving $\int \frac{x^3}{x^2-4} \ dx$ is a trig substitution.

A good first step in solving $\int \tan^{-1} x \ dx$ is to integrate by parts.

A good first step in solving $\int \frac{1}{x^2+4x+5} \ dx$ is completing the square.

If f is continuous and $\lim_{x\to\infty} f(x) = 0$ then $\int_1^\infty f(x) \ dx$ converges.

If f is even and $\int_0^\infty f(x) \ dx$ converges, then so does $\int_{-\infty}^\infty f(x) \ dx$.

If a_n converges, then $\lim a_n - a_{n-1} = 0$.

The harmonic series converges to zero.

If $\sum a_n$ converges to S then the sequence of its partial sums converges to S.

The series $1 - 1 + 1 - 1 + 1 - 1 + \ldots$ converges to 0.

Every increasing sequence that is bounded above converges.

BONUS (5 pts) Compute the sum of the series $1/2 + 2/4 + 3/8 + 4/16 + \ldots$. Hint: it is not geometric or telescoping, so it is not easy. But it *is related* to the series $1/2+1/4+1/8+\ldots$.