Exam III, 100 minutes, (Key) April 1, 2003

The average was about 64/100.

1) This was HW problem Ch 6.4-23 and was also on Exam II.

$$
\begin{aligned}
L &= \int_1^3 [1 + (f')^2]^{1/2}\ dx \\
&= \int_1^3 [1 + (x^2/2 - x^{-2}/2)^2]^{1/2}\ dx \\
&= \int_1^3 [x^4/4 + 1/2 + x^{-4}/4]^{1/2}\ dx \\
&= \int_1^3 x^2/2 + x^{-2}/2\ dx \\
&= x^3/6 - x^{-1}/2\ |_1^3 = 27/6 - 1/6 - 1/6 + 1/2 = 14/3
\end{aligned}
$$

Mistakes - mostly algebra and/or $f'$.

2) There are two good trig subns, and two good answers:
A) $u = \sec t$ leads to $(\sec^8 t)/8 - (\sec^6 t)/3 + (\sec^4 t)/4 + C$
B) $u = \tan t$ leads to $(\tan^6 t)/6 + (\tan^8 t)/8 + C$

3) Set $x = 3\sec u$, so $dx = 3\sec u \tan u\ du$. Get

$$
\begin{aligned}
\int 3\tan^2 u\ du &= 3\int \sec^2 u - 1\ du \\
&= 3\tan u - 3u + C \\
&= \sqrt{x^2 - 9} - 3\sec^{-1}(x/3) + C
\end{aligned}
$$

4) The partial fraction method gives $\int 1/x + 4/(x-4) = \ln|x| + 4\ln|x-4|$. Other methods are possible, but are generally harder.

5) Split the numerator into $(2x+4)+1$ and solve the 2 integrals (by a $u$ sub, and by completing-the-square) and get $\ln|x^2 + 4x + 5| + \tan^{-1}(x+2) + C$.

Another way (slightly harder, I think) is to c-the-square first and set $x + 2 = \tan u$ and go thru $\int \tan u + 1\ du$ etc.

6) $\lim_{t\to\infty} -xe^{-3x}/3 - e^{-3x}/9|_0^t = 1/9$ Using "lim" correctly and the IBP work were worth about 3-4 points each.

7a) (Advertised) See lecture notes or page 698.

7b) (Advertised) See lecture notes or page 686.

7c) See lecture notes.

8a) This was done in class and labelled "the trick". Do IBP twice to get: Answer $= e^x \sin x - e^x \cos x -$ Answer. Then solve for Answer.

8b) See Ex.6 on page 499.

9) FTTFT TFTFT

Bonus) Split the fractions up so that every numerator is 1 and regroup them into geo-series:

$$[1/2 + 1/4 + 1/8 + \ldots] + [1/4 + 1/8 + 1/16 + \ldots] + [1/8 + 1/16 + \ldots] + \ldots$$

$$= 1 + 1/2 + 1/4 + 1/8 + \ldots = 2$$