

Name

Show all your work and explain your reasoning. Don't use your own paper, a calculator or book. You may ask for paper or for clarifications.

- 1) [5pts] Find the average value of $f(x) = x^4$ on the interval $[0, 2]$.
- 2) [10pts] Approximate $\ln 2$ two ways (you may leave your answers as fractions):
- A) Give a Taylor series for $\ln(1 + x)$ and set $x = 1$ (this should produce a familiar series). Then find the 3rd partial sum. Give an error estimate for this answer.
- B) Give the integral definition of $\ln(x)$ and set $x = 2$. Estimate the integral using Simpson's rule with $n = 2$. Don't do an error estimate.

- 3) [5 pts each] Compute (or explain why no answer exists):

$$\int \tan^2 x \sec^2 x \, dx$$

$$\int x^2 e^x \, dx$$

$$\int_0^4 |x - 2|^{-1/2} \, dx =$$

$$\int_e^{+\infty} \frac{dx}{x \ln x}$$

- 4) [5pts] Use a famous power series to find the McLaurin series for $f(x) = x \sin(x^2)$.
- 5) [10pts] Find the interval of convergence of the series

$$\sum_{n=1}^{\infty} \frac{3^n x^n}{n^3}$$

- 6) [10pts] Find the area of the the region inside $r = 2 \sin \theta$ and outside $r = 1$. Begin by sketching the curves.
- 7) [10pts total] Determine whether each series converges or diverges (and explain briefly):

$$\sum_{n=1}^{\infty} \frac{3^n}{2^n + 4^n}$$

$$\sum_{n=1}^{\infty} \frac{n!}{e^{n^2}}$$

$$\sum_{n=1}^{\infty} \frac{(-1)^n n}{1+n^2}$$

8) [20pts] Answer True or False:

The shell method formula for volume is $V = \int_a^b 2\pi x f(x) dx$.

The formula for arc length is $L = \int_a^b \sqrt{1 + (f'(x))^2} dx$.

The integral $\int_1^{+\infty} 1/x^p dx$ converges if and only if $p > 1$.

There are exactly 8 points on the graph of $r = \cos(2\theta)$ where the tangent line is horizontal.

If $0 \leq a_n \leq b_n$ and $\sum b_n$ converges, then $\sum a_n$ converges.

The Mclaurin series for $\tan^{-1} x$ is $x - x^3/3 + x^5/5 + \dots$ but it doesn't converge for all real x .

The polar equations $r = \sin 4\theta$ and $r = \cos 4\theta$ have the same graphs.

A bounded increasing sequence must converge.

The Trapezoid rule, with $n = 20$, will produce an estimate for $\int_0^1 x^2 dx$ that is slightly too small.

The Trapezoid rule, with $n = 20$, will produce an estimate for $\int_0^{\pi/2} \cos(x) dx$ that is slightly too small.

9) [10pts] Choose ONE:

a) State and prove the integration formula for area in polar coordinates. Draw a picture showing a , b , etc., and use a \sum and a limit in your explanation.

b) Prove the FTC part I; that the derivative of $\int_a^x f(t) dt$ is $f(x)$.