Show all your work and reasoning for maximum credit. Do not use a calculator, book, or any personal paper. You may ask about any ambiguous questions or for extra paper. Hand in any extra paper you use along with your exam. These are 10 points each, except the T-F is 20pts, total.

1) Find a unit vector in the direction in which f **decreases** most rapidly at P, and find the rate of change of f at P in that direction; $f(x, y) = \cos(3x - y)$; $P(\pi/6, \pi/4)$.

2) Express the integral as an equivalent integral with the order of integration reversed; $\int_0^2 \int_0^{\sqrt{x}} f(x, y)\, dy\, dx$.

3) Evaluate the iterated integral by converting to polar coordinates;

$$\int_0^1 \int_0^{\sqrt{1-x^2}} (x^2 + y^2)\, dy\, dx.$$

4) Find an equation for the tangent plane and parametric equations for the normal line to the surface $z = 4x^3y^2 + 2y$ at the point $P(1, -2, 12)$.

5) True - False: Let T be the triangle with vertices at $(0, 0,)$ $(1, 1)$ and $(2, 0)$. Which of the following are equivalent to $\int\int_T x^2\, dA$? ['true' means 'equal'].

a) $\int_0^2 \int_0^{1-|x-1|} x^2\, dy\, dx$

b) $\int_0^1 \int_{y-2}^{y} x^2\, dx\, dy$

c) $\int_0^1 \int_y^{2-y} \int_0^{x^2} dz\, dx\, dy$

d) $x^2/2$

e) $\int_0^1 \int_0^x x^2\, dy\, dx + \int_1^2 \int_0^{2-x} x^2\, dy\, dx$

f) $7/6$

The rest of the true-false are not related to those above;

g) A 'Lagrange multiplier' is a special type of gradient vector.

h) The Jacobian of the transformation $u = x + y$, $v = x - y$, is a constant.

i) The Jacobian of the transformation $u = x^3 + y$, $v = x - \sin(y)$, is a constant.

j) The set where $1 \le x^2 + y^2 \le 5$ and $1 \le x \le 2$ is a polar rectangle.

6) Use spherical coordinates to find the volume of the solid bounded above by the sphere $\rho = 4$ and below by $\phi = \pi/3$.

7) Express the surface area as an iterated double integral in polar coordinates, and compute the area; the portion of the cone $z = \sqrt{x^2 + y^2}$ that lies inside the cylinder $x^2 + y^2 = 2x$.

8) Let G be the rectangular box bounded by $0 \le x \le \pi$, $0 \le y \le 1$ and $0 \le z \le \pi/6$. Evaluate the triple integral $\int\int\int_G xy\sin(yz)\ dV$.

9) Use Lagrange multipliers to find the maximum and minimum values of $f(x,y) = 4x^3 + y^2$ subject to the constraint $2x^2 + y^2 = 1$.

Bonus [about 5 points; E2, Ch 14.7]: Change variables, and use a Jacobian, to evaluate $\int\int_R \frac{x-y}{x+y}\ dA$, where R is bounded by $x-y=0$, $x-y=1$, $x+y=1$ and $x+y=3$. You can answer on the back.

Remarks and Answers: The average was about 70/100, with generally good results except on problems 1 and 7. The approx scale is:

A's 80 to 100
B's 70 to 79
C's 60 to 69
D's 50 to 59

Also, I added your 3 exams scores and estimated your current semester grade; see the upper right corner of your page one in red ink.

1) $\nabla f(P) = \sqrt{1/2}\ \langle -3, 1\rangle$. Normalize and change sign to get $\mathbf{u} = \sqrt{1/10}\ \langle 3, -1\rangle$. The rate is $-||\nabla f(P)|| = -\sqrt{5}$.

2) $\int_0^{\sqrt{2}} \int_{y^2}^2 f(x,y)\ dx\ dy$.

3) $\pi/8$

4) $\nabla(f(x,y) - z) = \langle 48, -14, -1\rangle$; the t.plane is: $48(x-1) - 14(y+2) - (z-12) = 0$; the normal line is:
$x = 1 + 48t$
$y = -2 - 14t$
$z = 12 - t$

5) TFTFT TFTFF

6) $\int_0^{2\pi}\int_0^{\pi/3}\int_0^4 \rho^2 \sin\phi\ d\rho\ d\phi\ d\theta = 64\pi/3$

7) $\int\int_R \sqrt{2}\ dA = \sqrt{2}\pi$. Note that $r^2 = x^2 + y^2$, so $f_x = r_x = x/r$ and $f_y = r_y = y/r$ so the integrand is $\sqrt{(x^2+y^2)/r^2 + 1} = \sqrt{2}$. Also, $\int\int_R\ dA$ is the area of a circle of radius 1, allowing a shortcut.

8) $\pi[\pi - 3]/2$

9) Most people set this up correctly and got to the eqns $3x^2 = \lambda x$, $y = \lambda y$ [which is worth about 6 points] but then got lost somewhere in the algebra. One fairly obvious continuation is $\lambda = 1$, $x = 1/3$, $y = \pm\sqrt{7}/3$; but there is also $y = 0$, $x = \pm\sqrt{1/2}$; and there is $x = 0$, $y = \pm 1$. The values from these 6 are: $4/27 + 7/9 = 25/27$ (occurs twice) and $\pm\sqrt{2}$ and 1 (twice). The max and min are $\pm\sqrt{2}$. See 13.9.7.

You don't need a calculator to see $\sqrt{2} > 1 > 25/27$. But if you get confused over something like this on an exam, make a guess, with a note, and you will not lose many points for it.

Bo) See Example 2 in 14.7.

Most of the problems on the exam came from the book; see 13.6 - 57, 13.7 - 5, 13.8 - 40, 13.9 - 7, 14.2 - 47, 14.3 - 27, 14.4 - 5, 14.5 - 9, 14.6 - 13 (maybe not all these, and in a different order).