

Name

Show all your work. Use the space provided, or leave a note. Don't use a calculator or your own extra paper. The problems are 10pts each.

1) Use Stoke's theorem to evaluate $\int \int_S (\text{curl} \mathbf{F}) \cdot \mathbf{n} \, dS$ where $\mathbf{F} = \langle 3y, -2x, xyz \rangle$ and S is the hemispherical surface $z = \sqrt{4 - x^2 - y^2}$ with upper unit normal vector.

2) Find the moment of inertia $\int \int_S x^2 + y^2 \, dS$ of the surface S with respect to the z -axis. Assume $\delta = 1$. S is the part of the plane $z = x + y$ that lies inside the cylinder $x^2 + y^2 = 9$.

3) Find a potential function for the vector field $F(x, y, z) = yz\mathbf{i} + xz\mathbf{j} + xy\mathbf{k}$. Explain your reasoning.

4) Sketch the region of integration. Reverse the order of integration. Evaluate the resulting integral. Do **not** compute the original integral, unless you want to check your answer.

$$\int_{-2}^2 \int_{x^2}^4 x^2 y \, dy \, dx$$

5) Determine whether the line L and the plane P are parallel or intersect. If they intersect, find the point of intersection.

$$L : x = 7 - 4t, y = 3 + 6t, z = 9 + 5t$$

$$P : 4x + y + 2z = 17$$

6) Answer True or False. You do not have to explain. Assume f is differentiable.

The differential of $f(x, y)$ is $df = f_x(x, y)\Delta x + f_y(x, y)\Delta y$

$$D_{\mathbf{u}}f(\mathbf{x}) = |\nabla f(\mathbf{x})|\mathbf{u}$$

If $f(x, y) \geq 0$ on a region R , then $\int \int_R f(x, y) dx dy \geq 0$.

If \mathbf{a} and \mathbf{b} are parallel, then $\mathbf{a} \times \mathbf{b} = 0$.

$(0,1)$ is an interior point of the ball $B = \{(x, y) : x^2 + y^2 \leq 1\}$.

7) Find the curvature κ for $y = \cos(x)$ at $(0,1)$. Use the formula for plane curves if you know it. If not, you may be able to work with the formula for space curves, $\kappa = |r' \times r''|/v^3$.

8) Use integration in polar coordinates to find the volume of the solid bounded by $z = 8 - r^2$ and $z = r^2$.

9) Use LaGrange multipliers to find the max and min values for $f(x, y, z) = xy + 2z$ subject to the constraint $x^2 + y^2 + z^2 = 36$.

10) Choose ONE.

A) State and prove the Divergence Theorem.

B) State and prove the Fundamental Theorem for Line Integrals.

Bonus (5pts)) Compute the integral $\int_0^{+\infty} e^{-x^2} dx$, as done in class, showing all work.