Combinatorics Homework 3, 4 and 5
S Hudson, 5/15 - 6/1/08

Quiz 2: Tuesday 5/20/08, will cover Chs. 2, 3 and 5.1-5.3. But most of the problems will resemble HW 2 problems, which only go through approx Ch 3.3. I hope Quiz 2 will be a little easier and faster than Quiz 1.

Practicing problems is the best way to study. But I'd also expect you to know the main formulas (eg Thm 2.2.1 and 3.2.2, etc - maybe you could even figure these out yourself?). Learn any formula that you might need, for example, to solve any HW problem. Be able to state Ramsey's theorem, and know what $r_t(q_1, q_2, \ldots q_k)$ means.

HW 3: This will not be collected - you do not have to write these problems up. But Quiz 3 may resemble these problems, so try to do them by Thursday 5/22/08 and let me know of any major problems you have with them. Remember that 5/26 is a holiday, so you may not have many chances to ask questions between 5/22 and Quiz 3 (on 5/27).

Ch. 3 - 21, 28, 30, 32, 36, 39, 41, 43, 47, 50, 54

Ch. 5 - 3, 5, 7, 9, 13, 15, 19, 23

Misc remarks on these problems:

For 3.32, there is a simpler answer than the one in the book, because an 11-permutation is really a 12-permutation; 12!/(3!4!5!).

For 3.39, the book gives a very useful hint (in the back).

For 3.50, assume the sides of the rectangle are rows or columns of the chessboard (can anyone solve it without this assumption ?).

Problem 3.54 is similar to one of the MIT problems mentioned in class.

Give a short proof of your answer (the recursive formula) in 5.3.

Likewise, explain 5.13 (there is a nice combinatorial proof of the answer).

I plan to do 5.20 in class, and 5.19 is similar.

Quiz 3: is Tuesday, 5/27/08, mainly on problems like HW3. Also on the lectures thru 5/22 and the reading of Ch 3, Ch 4.5 (but not covers or linear extensions) and Ch 5. There may be some overlap with Quiz 2 material (Ch3). You are not expected to memorize all the binomial identities in the book, but should know the main ones, and should be able to derive most of the others (see Ch5 HW).

I probably won't ask you to prove Sperner's theorem or Dilworth's theorem, but you should know the main ideas in those proofs (I might ask exercise 5.34, or to do one step of a proof, for example).

HW 4: Due Thursday, May 28

Ch. 4 - 36, 37, 49

Ch. 5 - 24, 25, 29, 31, 35, 38, 40, 46, 47

Ch. 6 - 1, 7, 11, 15

Remarks - 4.36 Hint: you can imagine a relation as a zero-one nxn matrix, if that helps.

- 4.37 You can find proofs like this in a Discrete Math book if you are completely stuck, but please don't just copy.

- 5.46 see page 150 (don't use 1+9).

Quiz 4: Tuesday, 6/3/08, will cover HW 4 and related sections; roughly 4.5 through 6.4. There might be a very easy Ch 7.1 problem. There may be one proof from this list -

1) The Inclusion/Exclusion Principle (I'll accept either Cor 6.1.2, with the proof given in class, or Thm 6.1.1 with the proof on page 163 - they are really the same).

2) Theorem 5.7.1 (if you are following the text, also explain the 'as already noted' step).

3) The Binomial Theorem (the second proof, eg the inductive proof).

I may give you some choice of proofs during the exam.

HW 5: Due Thursday, June 5.

Ch. 6 - 12, 16, 17, 21, 24ab, 28,

Ch. 7 - 1, 2, 3a, 8, 12, 16, 19abcd, 20, 23, 25, 28

Remarks: If you don't know how to use induction yet, you'll need to learn fast! I'm posting some sample answers to induction problems such as 7.3b and 7.19e (there should be a link to this on the syllabus). There are also 1 or 2 sample proofs on my Discrete Math exam, and you can probably find lots more in any Discrete Math textbook. For a little extra credit on HW 5, you can do some of the harder problems below (which I haven't tried yet). If you do any of these, leave me a note on the top of page one:

EC = 6.31, 6.32 (but the picture is wrong), 7.6, 7.22

Quiz 5: Tuesday, 6/10/08, will cover HW 5 and related sections; roughly 6.1 through 7.5 or 7.6 (but not 6.5 or 6.6). There may be one proof from this list -

1) The Chinese Remainder Theorem (not too likely)

2) Theorem 7.6.1 about the polygon.

3) Induction proofs similar to the homework.

HW 6: Due Thursday, June 12. This is the last one I will collect, but I may also assign some practice problems for the final exam.

Ch. 7 - 29 (and simplify all but d), 30a, 30c, 32, 33, 36, 37

Ch. 8 - 1, 2 (you can assume the x_{ij} are distinct), 6, 8, 21, 22a, 26, 27, 29

Do more of 7.30 as needed to master this method [but part b looks messy, and I haven't tried d,e,f].

My plans for the last two weeks include these sections: 8.1-8.4, (omitting multiplication schemes and Stirling numbers) and 9.1-9.4. We may have time for 1-2 other minor topics - you can send requests. Obvious possibilities are Ch 4.1-4.4, 6.6, 7.7, 8.2B, 8.5. Or, I could show you the Hudson Marriage Theorem (related to 9.3). Ch 8.2B includes some topics we've hit lightly already (counting onto functions and/or equivalence relations). It is likely that we can use 6/18/08 for review.

The Final: There will be no quiz or HW in the last week - just the Final Exam on Thursday, at the same time, same place, as our regular class meetings. About one-third of the final will be a review of topics from Quiz 1 through Quiz 5 (Chs 1-6 and early 7, not including the textbook proofs). About two-thirds will be on Chs 7.5 - 9.4. Review HW 6. Also, practice the problems from Ch.9 with answers or hints. To be more specific:

Ch. 9 - 1, 2, 3, 4, 7, 9, 10, 12, 13, 15, 20, 22, 23 and 27.

I've removed the domino problems, 5 and 17, but you should understand the connection with bipartite graphs, at least. I've left 20, 22 and 23 on the list, though I have not worked them out yet (actually, problem 20 seems very obvious, but maybe I'm missing something). I'll post an answer key ASAP, listed on the syllabus as a "key to HW7".

Proofs to Know: Thms 9.2.1 and 9.3.2.

The final exam will be only 75 minutes, so practice to perform algorithms and write proofs at a fair pace. The final may have an easy question about some topic from the 6/18/08 lecture, but if you pay attention that day, that should be enough. I will not start any new section of the book that day.

I may still allow a little extra credit until the final, but none afterwards. I plan to write Prof Brualdi to say his book is great, but also suggest re-wording some exercises. If you have any suggestions to include, please let me know ASAP.