MAS 3105 — Jan 21, 2003

Quiz I Key — Prof. S. Hudson

1) Use Gaussian elimination to put the following system into reduced row echelon form. Use matrix notation. You don't have to find the solution set.

$$\begin{aligned} x_2 + x_3 &= 1 \\ 2x_1 + 2x_2 + x_3 &= 4 \end{aligned}$$

Answer: Switch the 2 rows, divide the 1st by 2, and subtract the 2nd from the 1st:

$$\begin{pmatrix} 1 & 0 & -1/2 & | & 1 \\ 0 & 1 & 1 & | & 1 \end{pmatrix}$$

2) This augmented matrix is in RREF. Find the solution set, using α notation (if necessary) in your answer.

$$\begin{pmatrix} 1 & 2 & 0 & 1 & | & 5 \\ 0 & 0 & 1 & 3 & | & 4 \end{pmatrix}$$

Answer: $x_4 = \alpha,\ x_3 = 4-3\alpha,\ x_2 = \beta,\ x_1 = 5-\alpha-2\beta.$ So, $S = \{(5-\alpha-2\beta, \beta, 4-3\alpha, \alpha)\}$. [It is a good idea to check your answer. For example, set $\alpha = 1$ and $\beta = 2$ and check that (0,2,1,1) is a solution. This is more important in harder problems].

3) Answer each part with "True" or "False".

a) A 3x4 augmented matrix in RREF must have at least two leading 1's .
b) Gaussian elimination can change an inconsistent system into a consistent one.
c) Any two inconsistent 2x5 systems are equivalent.
d) An underdetermined system can be inconsistent.
e) If $AB = AC$ and $A \neq O$ (the zero matrix), then $B = C$.

Answer: FFTTF

SMALL BONUS: Justify your answer to the last True-False question.

Answer: Give a specific counterexample using 2x2 matrices. There are many possibilities, but A has to be singular. For example, set

$$A = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix} \quad B = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix} \quad C = \begin{pmatrix} 0 & 2 \\ 0 & 0 \end{pmatrix}$$