MAS 3105 Sept 5, 2008
Quiz I Prof. S. Hudson

1) Use Gaussian elimination to put the following system into reduced row echelon form. Use matrix notation. You don't have to find the solution set.

$$\begin{aligned} x_2 + x_3 &= 1 \\ 2x_1 + 2x_2 + x_3 &= 4 \end{aligned}$$

2) Label each system as underdetermined, overdetermined or square. Then describe how many solutions there are, and explain that briefly.

$$A = \left(\begin{array}{cccc|c} 1 & 2 & 0 & 1 & 5 \\ 0 & 0 & 1 & 3 & 4 \end{array}\right), \quad B = \left(\begin{array}{ccc|c} 1 & 2 & 0 & 5 \\ 0 & 0 & 0 & 1 \end{array}\right), \quad C = \left(\begin{array}{cc|c} 1 & 0 & 5 \\ 0 & 1 & 4 \end{array}\right)$$

3) Answer each part with "True" or "False".

a) A 3x4 augmented matrix in RREF must have at least one free variable.
b) Gaussian elimination can change an inconsistent system into a consistent one.
c) Any two consistent 2x5 systems are equivalent.
d) If $AB = AC$ and $A \neq O$ (the zero matrix), then $B = C$..
e) An overdetermined system can be consistent.

SMALL BONUS: Justify your answer to the last True-False question; part (e).

Remarks and Answers: The average grade was about 53/60; good! An approx scale is A: 55-60, B: 49-54, C: 43-48, D: 37-42. I expect the scale will come down by mid-semester.

1) Start by swapping the two rows. Please imagine a single vertical line after column 3.

$$\left(\begin{array}{ccc|c} 1 & 0 & -1/2 & 1 \\ 0 & 1 & 1 & 1 \end{array}\right),$$

2a) Underdetermined; infinite solution set, since it is consistent with a free variable.

2b) Underdetermined; no solutions, since it says $0 = 1$.

2c) Square; it has one solution (an obvious one), since it is in triangular form.

3) FFFFT; but I gave credit for either answer on the first statement.

Bonus: The usual method to show something exists, is to give an example. Try to find one that is as simple and clear as possible. One good answer is to write out $x_1 + x_2 = 1$ three times (and explain briefly that it is consistent and overdetermined).

I also gave credit for saying that any homogeneous overdetermined system must be consistent. But can an overdetermined system be homogeneous ? Of course it can, but giving a specific example would be slightly clearer.