

1) Find the solution set, using α notation, if necessary.

$$\begin{aligned}x_1 + 3x_2 + x_3 &= 2 \\x_2 - x_3 &= 4\end{aligned}$$

2) For each system, label it as underdetermined, overdetermined or square. Also, label it as RREF, REF (but not RREF) or neither. Then describe how many solutions there are, and explain that briefly. [total = 9 answers].

$$A = \left(\begin{array}{cccc|c} 1 & 2 & 0 & 1 & 5 \\ 0 & 0 & 1 & 3 & 4 \end{array} \right), \quad B = \left(\begin{array}{ccc|c} 1 & 2 & 0 & 5 \\ 0 & 0 & 0 & 1 \end{array} \right), \quad C = \left(\begin{array}{cc|c} 1 & 0 & 1 \\ 1 & 1 & 1 \end{array} \right)$$

- A)
- B)
- C)

3) Answer each part with “True” or “False”.

- a) An overdetermined system can be consistent.
- b) Gaussian elimination can change a consistent system into an inconsistent one.
- c) Any two consistent 2×5 systems are equivalent.
- d) Matrix multiplication is associative (eg, the associative property is true in this setting).
- e) Gaussian elimination can change a non-homogeneous system into a homogeneous one.

SMALL BONUS: Justify your answer to the last True-False question; part (e).

Answers and Remarks: Quiz I is intended to help you get off to a good quick start in this fast-paced course. The material is relatively easy and the Quiz should not seem very hard. The average grade was about 52 out of 60, based on grades over 40. There were several scores over 60, including two 64's. The (unofficial) scale for this quiz is a little higher than the one on the syllabus:

A's 55-60
B's 50-54
C's 45-49
D's 40-44

1) $S = \{(-10 - 4\alpha, 4 + \alpha, \alpha)\}$.

Common "mistakes": a) Doing GE. This is not needed since the system is already in REF. It is actually OK, but it often led to calc errors. b) Setting $x_2 = \alpha$. Since x_2 has a leading

one in its column, it is a lead variable, not a free variable. The free variable is x_3 , so let $x_3 = \alpha$. Again, it is debatable whether this is a serious mistake, but with linear systems, always aim for the standard method and the standard solution (you can be more creative later).

2A) underdetermined, RREF. It has infinitely many solutions, because a) it has a free variable, and b) it is not inconsistent.

2B) underdetermined, REF. It has no solutions because the bottom eqn says $0=1$. I did not accept 'because it is inconsistent' (this comment merely restates 'it has no solutions' with different words).

2C) square, neither. It has a unique solution $(1, 0)$ by a simple calculation (I also accepted more general explanations involving triangular form, etc. But saying 'it is square' is not enough).

In general, in problem 2, each error (or poor explanation) was marked 'X' and counted as -2 points. A '?' next to a not-quite-correct explanation counted as -1 point.

3) TFFTF

Bonus: The trivial solution works only for homogeneous systems, but GE can't change the solution set.

I gave credit for some other ideas on the Bonus, but long complex explanations usually got less credit than simple clear ones. I did not grade the Bonus if you marked 3e as True.