MAS 3105 Feb 4, 2003
Quiz II Key Prof. S. Hudson

1) Find an elementary matrix $E$ so that $EA = B$.

$$A = \begin{pmatrix} 1 & 1 \\ 2 & 0 \end{pmatrix} \qquad B = \begin{pmatrix} 1 & 1 \\ 3 & 1 \end{pmatrix}$$

**Ans:** Row 2 of $B$ = Row 1 of $A$ + Row 1 of $A$. So, we apply the same operation to the 2x2 identity matrix and get $E$ (which we should quickly check):

$$E = \begin{pmatrix} 1 & 0 \\ 1 & 1 \end{pmatrix}$$

2) Find the inverse of the matrix $A$ by using Gaussian elimination on an augmented matrix. **Check** by multiplying out $AA^{-1}$.

$$A = \begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 2 \end{pmatrix} \qquad \textbf{Ans}: \ A^{-1} = \begin{pmatrix} 2 & 0 & -1 \\ 0 & 1 & 0 \\ -1 & 0 & 1 \end{pmatrix}$$

3) Choose ONE of these to prove. You can answer on the back.

a) If $A$ and $B$ are symmetric 3x3 matrices, and $AB = BA$, then $AB$ is symmetric.
b) If $A$, $B$ and $C$ are 3x3 matrices such that $AB = C$ and $B$ is singular, then $C$ is also singular.
c) Prove that a square matrix $A$ is singular if $\det(A) = 0$.

**Ans:** These are HWs 1.3.25, 1.4.15 (see my Help pages) and part of Thm 2.2.2. Do *not* use Thm 2.2.3 in your answer, since it comes later in the course (avoid circular reasoning). For c), do *not* start by assuming $A$ is singular; that is the other part of Thm 2.2.2 (the converse part).