MAS 3105 Feb 2, 2004
Quiz 2 Key Prof. S. Hudson

1) [15pt] Find the determinants of these matrices. If possible, find quick ways to do them (without using the definition) and explain briefly.

$$A = \begin{pmatrix} 1 & 2 & 3 \\ 0 & 5 & 6 \\ 0 & 0 & 7 \end{pmatrix} \qquad B = \begin{pmatrix} 0 & 0 & 7 \\ 0 & 5 & 6 \\ 1 & 2 & 3 \end{pmatrix} \qquad C = \begin{pmatrix} 1 & 2 & 1 \\ 2 & 3 & 2 \\ 1 & 2 & 1 \end{pmatrix}$$

2) [10pt] Show how you would enter the matrix A from problem 1) into Matlab.

3) [15pt] Find the D^{-1} quickly (without GE) using clues hidden in the equation below. Think about how multiplication relates to rows and columns. Check your answer by multiplying out.

$$DF = \begin{pmatrix} 1 & 0 & 2 \\ 3 & 1 & 3 \\ 11 & 3 & 14 \end{pmatrix} \begin{pmatrix} 1 & -2 & 4 & 5 & 6 \\ 2 & 3 & 5 & -9 & -8 \\ 3 & 1 & 6 & -2 & -3 \end{pmatrix} = \begin{pmatrix} 7 & 0 & 16 & 1 & 0 \\ 14 & 0 & 35 & 0 & 1 \\ 59 & 1 & 143 & 0 & 0 \end{pmatrix} = G$$

4) [20pt] Choose ONE of these to prove. You can answer on the back.

a) If A is nonsingular then it is row equivalent to A^{-1}. [You can quote theorems or results from HW to give a very short proof]

b) If A is any mxn matrix, then $A + A^T$ is symmetric.

c) Prove this part of Thm 1.4.3: If A is row equivalent to I, then A is nonsingular.

Remarks and Answers: The average was about 40/60, or about 67 per cent, which is a little low for a Quiz 2. So, the scale is more generous, with A's starting at 50/60 etc.

The problems on this quiz were not very hard, but were *not* all similar to homework problems [especially part 3), which had only a 40 per cent success rate]. In general, the remaining quizzes will probably be more like the HW than this one was.

Answer 1): $\det(A) = 35$ (it is upper triangular, so 1x5x7).
$\det(B) =$ -35 ($B = EA$ where $\det E = -1$).
$\det(C) = 0$ (because two rows are the same).

Answer 2): $A = [1, 2, 3; 0, 5, 6; 0, 0, 7]$ (enter). The commas are optional, but the semicolon is not. See page 506 for another method.

Answer 3): We collect columns from F to get the answer:

$$D^{-1} = \begin{pmatrix} 5 & 6 & -2 \\ -9 & -8 & 3 \\ -2 & -3 & 1 \end{pmatrix}$$

Why? A matrix can be thought of a a list of its columns. For example, $G = [g_1, g_2 \dots g_5]$. The given equation shows that $D \cdot f_1 = g_1$, etc (five separate equations). Re-ordering three of these equations and ignoring the other two, we get $D[f_4, f_5, f_2] = [g_4, g_5, g_2] = [e_1, e_2, e_3] = I$. So, $D^{-1} = [f_4, f_5, f_2] =$ the answer above. For full credit, you had to check the answer by multiplying D^{-1} times D.

Answer 4a): By the TFAE thm, we know A is row eq to I. Since A^{-1} is nonsingular too, the same reasoning shows A^{-1} is also row eq to I. By the transitive property (which was a HW2 problem), A is row eq to A^{-1}. Done.

Remark: It is also possible to explain this using E_k 's, though the proof above is probably clearer.

b) Nobody chose this one, but the proof is just a short calculation: $(A + A^T)^T = A^T + (A^T)^T = A^T + A = A + A^T$. Done (this should be explained a little, but it's easy and I leave it to you).

c) See the textbook. Use E_k's and *explain the reasoning* correctly.