MAS 3105 — Sept 27, 2006

Quiz II and Key — Prof. S. Hudson

1) [10pts] Suppose A is a type I matrix (it swaps rows) and B is a type III matrix. What is $\det(ABBA)$? Explain.

2) [20pts] Find 2x2 matrices A, B and C such that: none of them are O (the zero matrix), none are the same, and $CA = CB$. [This is similar to a HW problem, but that had $AC = BC$ instead].

3) [10pts] Find three different examples of a 2x2 matrix A such that $AA = A$. Hint: at least two can be pretty simple, maybe even familiar.

4) [20pts] Choose ONE of these to prove. Use the back if you need more space. Use words and sentences and standard methods to completely explain your reasoning and your formulas.

a) If A and B are nonsingular they are row equivalent (you can use facts from the HW to prove this).

b) If A is nonsingular then det(A) is nonzero (This part of Thm 2.2.2 - don't just quote that theorem).

c) If A has two identical rows, then det(A) = 0 (use induction).

Answers: The average, among grades over 30, was about 44/60, which is a little low for Quiz 2. But problems that ask you to give examples can be hard, so I think this average is OK. The unofficial scale is:

A's = 52 to 60, B's = 46 to 51, C's = 40 to 45, D's = 34 to 39, F's = 00 to 33.

1) det(ABBA) = det(A)det(B)det(B)det(A) = (-1)(1)(1)(-1) = 1.

2) I hope you learned from the HW problem that C must be singular. Also, choosing most entries to be zero, so that $CA = CB = O$, is a good idea (but not required). One answer is:

$$C = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \quad A = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix} \quad B = \begin{pmatrix} 0 & 2 \\ 0 & 0 \end{pmatrix}$$

3) There are many *projection* matrices (this name should make more sense by Ch 4), and most are singular. For example:

$$A_1 = I = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \quad A_2 = O = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix},$$

$$A_3 = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}, \quad A_4 = \begin{pmatrix} 1/2 & 1/2 \\ 1/2 & 1/2 \end{pmatrix}.$$

4a) = HW 1.4.24b. It uses the TFAE thm and HW 1.4.24a.

4b) You can give the entire proof of Thm 2.2.2 if you want, but since this is only half the theorem, it is a little simpler to explain that $U = I$ (from the TFAE) and omit the singular case.

4c) Done in class. A full proof should include a basis step and an induction step (including the induction hypothesis). And a discussion of the minors used to compute det(A).