MAS 3105 May 18, 2006
Quiz II and Key Prof. S. Hudson

Suppose you are asked to find the inverse of a 2x2 matrix A, which appears below. And I do most of the calculation for you:

$$[A|I] = \begin{pmatrix} 1 & 1 & 1 & 0 \\ -2 & -1 & 0 & 1 \end{pmatrix} \to \begin{pmatrix} 1 & 1 & 1 & 0 \\ 0 & 1 & 2 & 1 \end{pmatrix} \to \begin{pmatrix} 1 & 0 & -1 & ? \\ 0 & 1 & 2 & 1 \end{pmatrix}$$

1) A is the first two columns of the first matrix. What is A^{-1} ? Check your answer.

2) The first "$\to$" is a G.E. step. Find the elementary matrix E_1 that does that step. Then find E_1^{-1}.

3) Express A^{-1} as a product of elementary matrices. (Write them out, in order).

4) You can use this calculation to get an LU factorization of A. What is L ? [hint: use part 2] What is U? Check that $A = LU$.

5) [20 pts) Choose ONE of these to prove, on the back. Use words and sentences and standard methods to completely explain your reasoning and your formulas.

a) If A and B are symmetric and $AB = BA$, then AB is symmetric.
b) If A is nonsingular then det(A) is nonzero.
c) If A is row equivalent to B then B is row equivalent to A.

Remarks + Answers: The average was about 45/60, based on the grades over 30. The unofficial scale is: A's = 52-60, B's = 46-51, C's = 40-45, D's = 34-39. The average grade for the semester now is approx 48 (based on the top 16 grades).

1) Look at the last two columns of the calculation, and finish it (the last operation subtracts row 2 from row 1). Then check that a) $AA^{-1} = I$ and b) $A^{-1}A = I$. Based on our discussion in class, I gave full credit if you only checked one of these, but would have given extra credit if anyone had done both, since that is technically still required (until we know more).

$$A^{-1} = \begin{pmatrix} -1 & -1 \\ 2 & 1 \end{pmatrix}, \quad \text{Check}: \ AA^{-1} = \begin{pmatrix} 1 & 1 \\ -2 & -1 \end{pmatrix} \begin{pmatrix} -1 & -1 \\ 2 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = I$$

2) E_1 is the right half of the matrix that appears after the first $\to$ (why?). Since E_1 is elementary, you can find its inverse with no work.

$$E_1 = \begin{pmatrix} 1 & 0 \\ 2 & 1 \end{pmatrix} \quad \text{and} \quad E_1^{-1} = \begin{pmatrix} 1 & 0 \\ -2 & 1 \end{pmatrix}$$

3) The original calculation can be summarized as $E_2E_1A = I$ (looking at the left halves), or by $E_2E_1I = A^{-1}$ (right halves). So $A^{-1} = E_2E_1$ and you only have to find the E_2 that

does the second $\rightarrow$ (mentioned in my answer to part 1). I think you should multiply them out to check your answer, but that was not required.

$$A^{-1} = E_2 E_1 = \begin{pmatrix} 1 & -1 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 2 & 1 \end{pmatrix}$$

4) In this example, $E_1 A = U$ is already upper tri (just one step), so set $A = E_1^{-1} U = LU$:

$$LU = \begin{pmatrix} 1 & 0 \\ -2 & 1 \end{pmatrix} \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 1 \\ -2 & -1 \end{pmatrix} = A$$

5) See the textbook, lectures and/or my help pages for these proofs. In grading them, I looked at the overall plan, the correctness of the calculations and logic, and for key explanations. For example, in part a), I looked for phrases like "by the definition of symmetric" and "$(AB)^T = B^T A^T$ by a theorem of matrix algebra" and so on. In part b), you should mention the TFAE theorem, and the def of row eq, etc.