MAS 3105 Feb 1, 2007
Quiz 2 and Key Prof. S. Hudson

1) [20pt] Find the determinants of these two matrices. If possible, find quick ways to do them (without using the definition) and explain briefly. Hint for A: if you swap two rows, the matrix becomes upper triangular.

$$A = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 0 & 0 & 6 & 5 \\ 0 & 5 & 6 & 7 \\ 0 & 0 & 0 & 4 \end{pmatrix} \qquad B = \begin{pmatrix} 1 & 2 & 4 \\ 2 & 4 & 8 \\ 4 & 8 & 16 \end{pmatrix}$$

2) [20pt] Solve for X given that $AX + B = X$ and

$$A = \begin{pmatrix} 0 & 1 \\ 0 & -1 \end{pmatrix} \qquad B = \begin{pmatrix} -2 & -2 \\ 6 & 8 \end{pmatrix}$$

3) [20pt] Choose ONE of these to prove. You can answer on the back.

a) If A is nonsingular then it is row equivalent to A^{-1}. [You can quote theorems or results from HW to give a very short proof]
b) Prove this part of Thm 1.4.2: If A is row equivalent to I, then A is nonsingular.

Remarks and Answers: The average score was 16+7+10=33 out of 60, which is pretty low for Quiz 2. Problem 2 was based on HW from Ch 1.3, and should have been pretty easy. Also, you should walk into each quiz ready for any advertised proofs such as Problem 3b), [Thm 1.4.2].

The unofficial scale is A's 45-60, B's 37-44, C's 31-36, D's 25-30, F's 0-24

1) det A = -120, det B = 0

2) From $(A - I)X = -B$, multiply by on the LEFT to get $X = -(A - I)^{-1}B$ = see below. If you multiplied on the right, and made no other mistakes, you should get the matrix Y instead (which got about 12 points of partial credit).

$$X = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix} \qquad Y = \begin{pmatrix} -2 & -2 \\ 6 & 7 \end{pmatrix}$$

If you solved for x_{11} (etc) *correctly* [one person did], I gave full credit. But in my opinion, this is an awkward method, very likely to produce errors. So, I was not too generous with partial credit for it (about 14/20 if there was a mistake). Whichever method you use, I'd strongly suggest checking your answer - which should be much easier than solving the problem.

3a) From the TFAE Thm, we know A is row equivalent to I, and the same logic applies to A^{-1}. From the HW, since A and A^{-1} are equivalent to the same matrix, they are also equivalent to each other.

3b) A is row equivalent to I. From the defn, we get $E_k E_{k-1} \dots E_1 A = I$, which we can write as $MA = I$. To show that $M = A^{-1}$ and complete the proof, we must check that $AM = I$. In my lecture, I left this step to you, but it should be included in a careful proof. It is not hard - solve for A in the first equation, and get $A = E_1^{-1} E_2^{-1} \dots E_k^{-1}$. Then write out AM in terms of the E's and check that they all cancel out to I.

The proof in the text is a bit glib, but it's OK.