MAS 3105 Sept 19, 2008
Quiz 2 Prof. S. Hudson

1) [20pt] Find the determinants of these matrices. If possible, find quick ways to do them (without using the definition) and explain briefly.

$$A = \begin{pmatrix} 1 & 2 & 3 \\ 0 & 5 & 6 \\ 0 & 0 & 7 \end{pmatrix} \qquad B = \begin{pmatrix} 0 & 0 & 7 \\ 0 & 5 & 6 \\ 1 & 2 & 3 \end{pmatrix} \qquad C = \begin{pmatrix} 1 & 2 & 1 \\ 2 & 3 & 2 \\ 1 & 2 & 1 \end{pmatrix}$$

2) [20pt] Solve for X given that $AX + B = X$ and

$$A = \begin{pmatrix} 0 & 1 \\ 0 & -1 \end{pmatrix} \qquad B = \begin{pmatrix} -2 & -2 \\ 6 & 8 \end{pmatrix}$$

3) [20pt] Choose one of these HW proofs; you can answer on the back:

A) If A is row equivalent to both B and to C, then B is row equivalent to C.

B) Suppose these 3 matrices are all nxn, and $AB = C$. Prove: if B is singular, then C is, too.

Bonus: (about 5 points) Almost the same as part 3B, But prove: if A is singular, then C is, too. Hint: transpose.

Remarks and Answers: I was short of time, and based this Quiz on one from Spring 2007. The average was about 47/60. The scale for this quiz is: A-'s start at 54, B-'s at 48, C-'s at 42, etc).

1) det A = 35, det B = -35, det C = 0.

2) From $(A-I)X = -B$, multiply by on the LEFT to get $X = -(A-I)^{-1}B =$ see below. If you multiplied on the right, and made no other mistakes, you should get the matrix Y instead (which got about 12 points of partial credit).

$$X = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix} \qquad Y = \begin{pmatrix} -2 & -2 \\ 6 & 7 \end{pmatrix}$$

I'd strongly suggest checking your answer - which should be much easier than solving the problem.

3) These were HW problems from Ch 1. I already have answers or hints posted on this site, but if you need more help, you can always see me.

Bonus: Suppose A is singular and $C = AB$. Then A^T is singular, because det A^T = det $A = 0$ [or, we could justify this step without determinants, using Exercise 17 of Ch 1.3).

So, $B^T A^T$ is singular (by problem 3b above, since the A^T is on the right side). So, the transpose of this is singular, namely AB. Done.

Note: Transposes have many purposes, and this proof shows one of them - to swap the left and right factors in a product.