MAS 3105 AM: Jan 31, 2008
Quiz 2 and Key Prof. S. Hudson

Note on problem 1: 'Find B' means - write out all 9 entries of B. 'Check' usually means - show me the calculation on paper. For 1c and 1d, I'll accept a short proof instead, if you prefer. Parts 1a - 1d are closely related, so do them in order, and you should not need any trial and error. A transpose might help with one part.

1) [10pts each part] 1a) For the matrix A given below, check that $A^3 = O$. Such a matrix is called *nilpotent.*

$$A = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix}$$

1b) Find a nonzero 3x3 matrix B, so that $AB = O$ (for the same A as above).

1c) Find two nonzero 3x3 matrices C and D so that $C - D = A$. Then, check that $CB = DB$.

1d) Find nonzero 3x3 matrices E, F and G so that $EF = EG$. Check.

2) [20pts] Choose ONE of these to prove. Use words and sentences and standard methods to completely explain your reasoning and your formulas. Some of these are just parts of theorems we did in class. If so, you are NOT allowed to simply quote the theorem! You ARE allowed to use previous theorems, definitions and/or previous HW.

a) If A is square, and the system $Ax = 0$ has only the trivial solution, then A is row equivalent to I.

b) If A is nonsingular then det(A) is nonzero.

c) If A is row equivalent to B, then B is row equivalent to A.

Bonus (5pts): Your first MATLAB problem refers to the matrix $(A' * B')'$. This notation is a bit different from that used in class. Show me that you understand it by simplifying this expression.

Remarks and Answers: The average was about 52/60 again, so the unofficial scale is the same as for Quiz 1 (A-'s start at 55, B-'s at 49 etc).

The main step of 1a) is to find A^2 (then check $A(A^2) = O$):

$$A^2 = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

1b) Set $B = A^2$ (but other answers are possible).

1c) Set $D = I$ and $C = A + I$, for example (or set $D = A$ and $C = 2A$, or etc). You can check by multiplying out CB and DB. Or, just explain that $AB = O$ implies $(C - D)B = O$, so $CB - DB = O$ and $CB = DB$.

1d) One simple answer is to transpose the previous equation and get $B^tC^t = B^tD^t$. So, set $E = B^t$ and $F = C^t$ and $G = D^t$. But most people found other examples, perhaps by trial and error.

I forgot to require that $F \neq G$, as in the similar HW problems (which prevents super-simple answers like $E = F = G = I$). If you got the spirit of the problem, and made $F \neq G$, I gave +1 point extra credit.

2) See text or lectures. As I type this, I've only graded the "b"s. Several people started with some statement like $U = I$. But how will the reader know what you mean by U ? And why is this mysterious matrix U equal to I ? You may assume the reader knows the vocabulary in the problem, but should not assume they already know the proof! You must introduce any new matrices/etc to the reader. For example, you could start by saying that A is row equivalent to some matrix U in RREF (this uses well-known results earlier in Ch1). Even better, quote the TFAE theorem, and just say that A is row equivalent to I. There is really no need for the letter U since you are only proving one direction of the usual theorem.

Bonus: A' means A^t, and $*$ means multiply, so it simplifies to BA.