MAS 3105 PM: Jan 31, 2008
Quiz 2 and Key Prof. S. Hudson

For problems 1-4 (which are 10 points each) let:

$$A = \begin{pmatrix} 1 & 1 \\ 1 & 2 \end{pmatrix}$$

1) Find an elementary matrix E so that $EA = U$ is upper triangular.

2) Find a lower triangular matrix so that $A = LU$ [where U is the same as in part (1)].

3) Find A^{-1}.

4) Find X, so that $AX - A = X$ (then turn the page for problem 5).

5) [20pts] Choose ONE of these to prove. Use words and sentences and standard methods to completely explain your reasoning and your formulas. You are allowed to combine previous theorems, definitions and/or HW. But do NOT simply quote the theorem/HW this comes from! - or use any later results.

a) If A and B and AB are all symmetric, then $AB = BA$.

b) If A and B are nonsingular, then $\det(AB) = \det\,(A)\,\det(B)$.

c) If A is nonsingular then $(A^t)^{-1} = (A^{-1})^t$.

Remarks and Answers: The average was again approx 52/60, so Quiz 2 has the same scale as Quiz 1 (A's start at 55, and each letter below that drops 6 points). I've combined the answers to 1) thru 3):

$$\text{Let } E = \begin{pmatrix} 1 & 0 \\ -1 & 1 \end{pmatrix} \quad \text{so, } U = EA = \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}$$

$$L = E^{-1} = \begin{pmatrix} 1 & 0 \\ 1 & 1 \end{pmatrix} \quad \text{and} \quad A^{-1} = U^{-1}E = \begin{pmatrix} 2 & -1 \\ -1 & 1 \end{pmatrix}$$

4) First, the algebra: $AX - X = A$, so $(A - I)X = A$ and $X = (A - I)^{-1}A$.

$$X = \begin{pmatrix} 0 & 1 \\ 1 & 1 \end{pmatrix}$$

Usually, a mistake like $X(A - I) = A$ would be more serious, but by a coincidence (which I did not intend), $X = A - I$ and $(A - I)X = X(A - I)$.

5) See the text or lectures. Most people chose a) and did OK on it. By luck, A and X from part 4) fit the hypotheses of 5a), so $AX = XA$.