MAS 3105 May 14, 2009
Quiz 2 and Key Prof. S. Hudson

1) If possible, find a nonzero 2x2 matrix A such that $A^2 = O$ (the zero matrix). If it is not possible, explain why not. Note: *nonzero* means at least one entry of A is not zero.

2) Write $\mathbf{v}$ as a linear combination of $\mathbf{u}$ and $\mathbf{w}$. For maximum credit, solve this using a reliable method (guessing the answer may only get partial credit).

$$\mathbf{v} = \begin{pmatrix} 1 \\ 2 \end{pmatrix} \quad \mathbf{u} = \begin{pmatrix} 2 \\ 1 \end{pmatrix} \quad \mathbf{w} = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

3) Choose ONE of these to prove. You can answer on the back.

a) If A is a symmetric nonsingular matrix, then A^{-1} is also symmetric [if you know a formula for the transpose of A^{-1}, you can use it without proving it].

b) Prove this part of the TFAE theorem: If A is row equivalent to I, then A is nonsingular.

Remarks and Answers: Average 44/60. A's = 50-60, B's 44-49, C's 38-43, D's 32-37.

1) There are many examples (all singular, of course), but several people used this one:

$$A = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}$$

2) [first version] There were two versions of the quiz. The first one shows the correct year, 2009, in the upper right corner. For that version, the answer to 2) is:

$$\mathbf{v} = -\mathbf{u} + 3\mathbf{w}$$

You can get this by solving a linear system, with GE, for example. For full credit, write the answer in the correct form, an equation similar to the one above. It is OK to use notation such as $c_1 = -1$, or $x_1 = -1$, in your calculations, but these stop a bit short of the correct form (these formulas were good for about 18 out of 20 points, though). In other cases, the notation was so bad that the answers were hardly recognizable, and those got less credit.

2) [second version] This version had an intentional typo - the year shows as 2008. Problem 2 was similar, except that $\mathbf{u} = [3, 2]^T$ and the answer is:

$$\mathbf{v} = -\mathbf{u} + 4\mathbf{w}$$

3a) $(A^{-1})^T = (A^T)^{-1} = A^{-1}$ (explain each step briefly, and why this proves 3a)

3b) See the text or lectures.