MAS 3105 Feb 18, 2003
Quiz 3 Prof. S. Hudson

1) Answer True or False:

If A is row equivalent to B, then det $A =$ det B.

If A is nonsingular, then A^T is nonsingular.

If A is $m \times n$ then $N(A)$ is a subspace of R^n.

For all 5x5 matrices B and C, det (BC) = det (CB).

If A is nonsingular then its columns are linearly independent.

Answer: FTTTT

2)*Short* answer! Label your answers clearly: a) Find det $(A^T A^{-1})$, b) Find det (adj$A$), c) Find det$(3A)$. Where:

$$A = \begin{pmatrix} 1 & 3 & 5 \\ 0 & 2 & 3 \\ 0 & 0 & 1 \end{pmatrix}$$

Answer: After noting that det $A = 2$, you can ignore A, and use theorems. You don't have to compute A^{-1} etc. For a), use $2 \times \frac{1}{2} = 1$. For b), start with A adj$A =$ det A I, and take a det of both sides;

$$\det A \det(\text{adj}A) = (\det A)^3 \ \det I$$

so that det(adj$A$) $= 8/2 = 4$. It is also OK to compute adj A, which isn't too hard in this example.

For c), $\det 3A = 3^3 \det A = 27 \times 2 = 54$.

3) Choose ONE of these to prove. Assume A and B are nxn matrices.

a) If $S = \{v_1, v_2 \ldots v_n\}$, then span$(S)$ is a subspace of V.
b) For all nxn matrices, $\det(AB) = \det(A)\det(B)$.
c) If A has two identical rows, then $\det(A) = 0$. (Use induction. Possible +2 bonus for choosing this one!).

Answer: Parts a) and b) are in the text, though the proof of b) on page 112 has only minimal explanation.

3c) This is HW 2.1-10. Don't include specific examples. Do include a basis step ($n = 2$) and an induction step ($n > 2$). You can then expand on row 1 *if* it is not one of the two identical rows (you cam "assume" this, but say so). Explain briefly the calculation that leads to 0.