MAS 3105 Feb 16, 2004

Quiz 3 Key Prof. S. Hudson

1) Use Cramer's Rule to solve for x_2. Show all your work clearly [and check your answer if you have time].

$$2x_1 + 3x_2 = 12$$
$$3x_1 + 2x_2 = 13$$

2) Suppose that A and B are 3x3 matrices. Answer True or False:

If A is row equivalent to B, then det $A =$ det B.

If B is row equivalent to a nonsingular matrix, then det (B) is nonzero.

If det $(2A) = 8$ then det $(A^{-1}) = 1$.

If det $(A) = 1$ then $A^{-1} =$ adj A.

The set of 2x2 matrices, denoted R^{2x2}, is a vector space.

3) Choose ONE of these to prove. Assume A and B are nxn matrices. You can answer on the back.

a) If A is square with two identical rows, then det $A = 0$. [Use induction.]

b) State and prove Cramer's Rule.

c) If $S = \{v_1, v_2 \dots v_n\}$, then span$(S)$ is a subspace of V [Mention all 4 parts of the definition of subspace, and prove parts 3 and 4 carefully].

Key: The average was about 47/60, or about 78 per cent - pretty good. Generally the scores were high on problems 1 and 2.

1) Answer: $x_2 = 2$ (and $x_1 = 3$, for checking purposes).

2) Answer: FTTTT

3) Answer: Part a) is HW 2.1.10, and the others are textbook proofs.

Remark on a): If you did the 2x2 case and included an equation like $A_{11} = 0$, then you probably had the right idea. But for full credit, you needed to use induction correctly (as in the proof of thm 2.1.2, for example). Include phrases like "the result holds if $n = 2$" and "Assume that the result holds for all kxk matrices, and (etc)". In the induction step, you should *not* expand along one of the two identical rows.

Remark on c): If your answer wasn't prefect, I tried to give partial credit for using the proper definitions of "subspace" and "span". Also, I wanted you to include a little more than appears on page 139, such as -

1) Span $(S) \subseteq V$, because any $v \in$ span (S) is a linear combination of vectors in V, so that $v \in V$.

2) Clearly, span $(S) \neq \emptyset$. For example, the trivial linear combination shows that the zero vector is in span (S). [for parts 3) and 4), see page 139].