MAS 3105 May 26, 2005
Quiz 3 and Key Prof. S. Hudson

Name

Show all your work and reasoning for maximum credit. If you continue your work on another page, be sure to leave a note. Do not use a calculator, book, or any personal paper. You may ask about any ambiguous questions or for extra paper. If you use extra paper, hand it in with your exam.

1) Answer True or False:

If A is row equivalent to B, then det A = det B.
If A is a nonsingular 3x3 matrix, then its columns form a basis of R^3.
P_2 is a subspace of P_3. (P_3 = the set of poly's of degree < 3).
Every square matrix is row equivalent to the identity or to a matrix with a row of zeroes.
det $(A^T A)$ is always greater than or equal to 0.

2) Answer all three parts, based on the matrix A given below.

a) Find adj A.
b) Use the answer to a) to find A^{-1}. [*Don't* use G.E.]
c) Find det A^{-1} [Try to do this without using the answer to b)]

$$A = \begin{pmatrix} 1 & 3 & 5 \\ 0 & 2 & 4 \\ 0 & 0 & 3 \end{pmatrix}$$

3) Choose ONE of these to prove. Assume that A and B are square nxn matrices. For maximum credit, explain every step clearly.

a) If $AB = I$, then $BA = I$. [Do not assume (without proof) that A has an inverse. You may use thms from Chs 1-2].
b) If A is upper triangular, then det $A = a_{11}a_{22}\dots a_{nn}$. [Use induction. Possible bonus for choosing this one]
c) If A is nonsingular, then det (AB) = det A det B (prove this part of Thm 2.2.3, and explain each step).

Answers and remarks: The average was about 53/60.

1) FTTTT The grades were very good on this one. Maybe I need a new batch of TF statements for my quizzes!

2a) Compute the cofactors, including minus signs, and transpose.

b) Divide each entry by det $(A) = 6$. I did not ask you to check your answer to 2b by multiplying, but it is a good idea. I take off a bit more for errors when checking the answer is this easy to do.

c) $(\det(A))^{-1} = 1/6$.

$$\text{adj } A = \begin{pmatrix} 6 & -9 & 2 \\ 0 & 3 & -4 \\ 0 & 0 & 2 \end{pmatrix}$$

3) Most of these are already in the lectures, book or my help pages. Some comments -

a) First, show that det $A \neq 0$ so that you can use A^{-1} [and its definition] in the rest of your proof, which should be pretty easy.

b) Some common mistakes were - not explaining the plan (not stating the induction hypothesis, etc) and notational problems (often so severe that the idea got lost).

c) I assumed you would use the proof from my lectures, which involves factoring A into E's. In that proof, it does not matter whether B is nonsingular. You should mention the theorem that det $(EB) = \det E \det B$ in your explanation.

In my opinion, the textbook proof is harder to explain. If you use it, you need to discuss two cases - when B is singular, or not. And you need to talk about det $(BE) = \det B \det E$.