MAS 3105 May 25, 2006

Quiz 3 and Key Prof. S. Hudson

1) Find $\det(A)$, if A is 3x3 and it factors into

$$A = LU = \begin{pmatrix} 1 & 0 & 0 \\ l_{21} & 1 & 0 \\ l_{31} & l_{32} & 1 \end{pmatrix} \begin{pmatrix} u_{11} & u_{12} & u_{13} \\ 0 & u_{22} & u_{23} \\ 0 & 0 & u_{33} \end{pmatrix}$$

2) Which of these are spanning sets of R^3? Justify your answers.

a) $\{(1,0,0)^T, (1,1,0)^T, (1,1,1)^T, (2,2,2)^T\}$

b) $\{(1,1,0)^T, (2,2,0)^T, (1,1,1)^T, (2,2,2)^T\}$

3) Choose ONE of these to prove (on the back is OK). Assume A and B are nxn matrices.

a) Show that if $AB = I$, then $BA = I$.
b) If L is a list of vectors in V, then span(L) is a subspace of V.
c) If A has two identical rows, then det $(A) = 0$ (use induction).

Bonus (5pt): Suppose $A = [1\ 1\ 1; 2\ 2\ 2; 3\ 3\ 3]$ (notation as in MATLAB, etc). Without doing any numerical work, predict the answer to $A * \mathrm{adj}A =$

Remarks + Answers: The average was about 50/60. Problem 1) was pretty easy. Problem 2 was harder, mainly because it came from Ch 3.2, not in HW3.

1) $\det(A) = \det(L)\ \det(U) = u_{11}u_{22}u_{33}$, since they are triangular. This shows another possible use of the LU factorization.

2a) Yes. We know that 3 vectors are usually enough to span R^3. The first three on this list form a nonsingular matrix, so those three alone span R^3 (and the fourth doesn't hurt). [Of course, you have to be a little bit careful - this reasoning does not work if you pick the *last* three vectors].

2b) No. One very careful explanation method is to mimic Example 11d (page 130-131), *including enough words.* I accepted more casual explanations such as, "Since v_2 is a scalar multiple of v_1 and v_4 is a scalar multiple of v_3, we really have only two vectors, which are not enough to span R^3". A faster way to say it is: dim (span(L)) $\leq 2 < \dim(R^3)$, so span $(L) \neq R^3$ (but this vocabulary is from Ch 3.4).

3) These are in the text and/or lecture notes. Rks on a); First use $\det(AB)$ to show A is nonsingular. *Then* you can use A^{-1} to do the simple algebra. On b); I really wanted you to show [or at least mention] that span (L) contains the zero vector, and is a subset of V, but our text does leave out those minor steps. On c); Probably the hardest of the 3.

Bonus: From Ch 2.3, the answer is $\det(A)I = O$ (the 3x3 zero matrix).