MAS 3105 October 3, 2008
Quiz 3 and Key Prof. S. Hudson

1) (30 pts) Answer True or False:

The set of symmetric 2x2 matrices, denoted R^{2x2}, is a vector space.

If A is row equivalent to B, then $\det A = \det B$.

adj $(A^T) = (\text{adj } A)^T$

If A is nonsingular, then adj $(A^{-1}) = (\text{adj } A)^{-1}$

$\{(x_1, x_2) : x_1 + 3x_2 = 0\}$ is a subspace of R^2.

$\{(x_1, x_2) : x_1 \geq 0\}$ is a subspace of R^2.

2) (10 pts) Suppose A is singular. What can you say about $A\cdot$ adj A ? (explain briefly)

What about (adj $A)A$? (explain briefly)

3) (20 pts) Choose ONE of these to prove. Assume A and B are nxn matrices.

a) If $AB = I$, then $BA = I$.

b) Use induction to prove that if A has two identical rows, then det $A = 0$.

c) $\det(AB) = \det(A)\det(B)$ [you can use facts about E's, HW results, and any previous theorems. But prove all the cases/lemmas/etc of Thm 2.2.3]

Bonus (about 5 points; from exercise 2.2.19): How many additions are required to compute the determinant of a 5x5 matrix using cofactors ? Explain.

Remarks and Answers: Your quiz was graded first by our student grader in red ink. I reviewed/adjusted) it in blue. The average was approx 50, so the A-'s start at about 55, and each letter below that spans 6 points (eg 10% of 60). In past semesters, the scale has usually come down towards the one in the syllabus, starting around Quiz 4.

1) There was a typo in the first part (I inserted "symmetric" into a previous statement, but forgot to remove the phrase "denoted R^{2x2}"). I intended it to be TRUE, but since it is flawed, I gave credit for either answer. For the other parts: FTTTF

2) a) From Ch 2.3, it is (det A) I = O (zero matrix).

b) Also O. There are several possible explanations. The Crazy Mistake Lemma is one good way to do it, but the transpose formula in the True-False above might work instead. Don't explain using A^{-1}, which doesn't even exist.

3) a) Use det's to show A^{-1} exists. Use that to show $B = A^{-1}$. The rest is easy.

b) The proof should have a basis step plus an induction step (which should include the induction hypothesis clearly). One key point is that each minor will have two identical rows (assuming the proof has been organized well).

c) This is in the book, and I expect a proof similar to that, with two cases and a calculation using E's. In the book, the cases are based on whether B is singular (OK, but explain the part about det $AE =$ det A det E). In class, the cases were based on whether A is singular; then you don't have to explain any det $AE =$ step (but must explain why AB is singular in one of the cases).

Bonus: see exercise 2.2.19.