MAS 3105 May 21, 2009
Quiz III and Key Prof. S. Hudson

1) [15pt] Apply the elimination method to compute det A (apply GE to simplify it, perhaps to triangular form, and then ask yourself if/how that affected the det):

$$A = \begin{pmatrix} 0 & 3 & 1 & 1 \\ 1 & 1 & 1 & 3 \\ 0 & 0 & 2 & 2 \\ -1 & -1 & -1 & 2 \end{pmatrix}$$

2) [5pt] How do you tell MATLAB to create a random 5x5 matrix ? (show me what you'd type onto the command line).

3) [20pt] True-False. You can assume all the matrices are square here, and in problem 4 below.

The set $S = \{[x_1, x_2]^T : x_1 + x_2 = 10\}$ is a subspace of R^2.

The set of symmetric 2x2 matrices is a subspace of R^{2x2}.

$\det(A^k) = (\det A)^k$

If A and B are row equivalent, then they have the same determinant.

If A is singular, then A adj $(A) = O$ (the zero matrix).

4) [20pt] Prove ONE: You can answer on the back.

a) Prove that if A is singular then adj(A) is also singular.

b) Suppose A and B are both nxn. Prove that if $AB = I$, then $BA = I$. Do not assume A is nonsingular (it might be true, but you'll have to prove it, if you want to use it).

c) Use induction to prove that if A is upper triangular, then det $A = a_{11}a_{22} \dots a_{nn}$.

d) Prove that $\det(AB) = \det(A)\det(B)$.

Remarks and Answers: The average was about 47 out of 60. Not bad. The unofficial scale for this quiz is

A's = 52-60, B's = 46-51, C's = 40-45, D's = 34-39.

Based on new info, the average for Quiz 2 has come down about 3 points, so you can adjust the scale on that quiz about 3 points in your favor. Remember that all these scales are approximations, partly based on my guesses about who will drop the class or drop certain quiz grades.

1) -30. Apply two GE steps, types I and III, to get an upper triangular matrix T with det $T = 30$. Type I's change the sign. It is possible (but not necessary) to GJ it all the way to RREF.

In class, I did not emphasize the method of elimination to find det's. But from the reading and HW, you can see that it is more efficient than cofactors or Cramer's Rule for most matrices. Efficiency can be important in very large real-life problems, but is generally beyond the scope of a first course in Linear Algebra.

2) rand(5)

3) FTTFT

4a) From Ch2.3, we know A adj $A =$ det $A\ I = O$ (since A is singular its det is 0). The rest is a proof-by-contradiction. If adj A were nonsingular, we could use its inverse to solve for A and get $A = O$. But then adj $A = O$ too (using the defn of adj), a contradiction. This shows adj A must be singular.

4b) Assume $AB = I$. By taking det of both sides, we see that det A cannot be zero, so A^{-1} exists. We can multiply it on both sides to get $B = A^{-1}I = A^{-1}$. So, $BA = I$.

4c) If you are having problems with induction, see me, or a Discrete Math book, or Velleman's book. There are also several examples worked out in our text and on this web site.

4d) See lecture notes. The most common problem was not writing enough, especially in these steps:

$$\begin{aligned}\det(E_k E_{k-1} \ldots E_1 B) =& \det(E_k)\det(E_{k-1} \ldots E_1 B)\\ =& (etc)\\ =& \det(E_k) \ldots \det(E_1)\det(B)\end{aligned}$$

based on the formula from class, that $\det(EB) = \det(E)\det(B)$ whenever E is elementary.

If you used the book's proof, you need $\det(BE) = \det(B)\det(E)$ instead. This is a bit harder to explain (which is why I prefer my proof).