MAS 3105 Feb 27, 2003
Quiz 4 Key Prof. S. Hudson

1) [40 points] Set $B = \{\mathbf{v_1}, \mathbf{v_2}, \mathbf{v_3}, \mathbf{v_4}\} = \{(1,0,0)^T, (1,0,1)^T, (0,1,1)^T, (0,0,1)^T\}$. So, B is a set of 4 column vectors in R^3. Answer, and explain briefly:

a) Is B a spanning set of R^3?

b) Is B linearly independent?

c) Let $C = \{\mathbf{v_1}, \mathbf{v_2}, \mathbf{v_4}\}$. Is C a basis of R^3?

d) Let $D = \{\mathbf{v_1}, \mathbf{v_2}, \mathbf{v_3}\}$. Show that D a basis of R^3?

e) Find the transition matrix from D to the standard basis of R^3.

2) [20 points] Choose ONE of these to prove.

a) Show that a nonempty subset of a linearly independent set of vectors $\{\mathbf{v_1}, \mathbf{v_2}, \ldots \mathbf{v_n}\}$ must also be linearly independent. [Use the definition of l.i.].

b) Show that if $\{\mathbf{v_1}, \mathbf{v_2}, \ldots \mathbf{v_n}\}$ is a basis of V, and $\mathbf{v} \in V$, then $\mathbf{v}$ can be written uniquely as a linear combination of $\{\mathbf{v_1}, \mathbf{v_2}, \ldots \mathbf{v_n}\}$.

Answers: [see also Quiz 4 of Summer 2000]. I gave 4 points for a correct 'yes' or 'no' and 4 points for the explanation in 1abc. Other explanations are possible:

1a) Yes, because even the first 3 vectors span R^3 (see 1d).

1b) No, 4 vectors in R^3 must be LD.

1c) No, the set is LD since $v_1 + v_4 - v_2 = 0$ (or explain with a determinant).

1d) Combining the vectors into a matrix A, we see det $A = -1 \neq 0$, so the columns form a basis of R^3.

1e) It is the matrix A used in 1d). Do NOT compute its inverse!

2a) This was done in class (and probably also appears in my help pages). A good proof should contain the key equation below (or the equivalent):

$$c_1\mathbf{v_1} + c_2\mathbf{v_2} + \ldots + c_j\mathbf{v_j} + 0\mathbf{v_{j+1}} + \ldots + 0\mathbf{v_n} = \mathbf{0}$$

2b) This is essentially the first half of Thm 3.3.2 (page 151). The main goal is to prove 'uniquely', but you should also mention that v is a LC of the other vectors (since they are a basis, they span $V \ldots$).