MAS 3105 Feb 25, 2004
Quiz 4 Key Prof. S. Hudson

1) The matrix A is row equivalent to U (see below). Find a basis for $N(A)$, and explain briefly how you know it is a basis.

$$A = \begin{pmatrix} 1 & 3 & 0 & 9 & 4 \\ 1 & 1 & 4 & 19 & 2 \\ 1 & 1 & 2 & 11 & 2 \end{pmatrix} \qquad U = \begin{pmatrix} 1 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 3 & 1 \\ 0 & 0 & 1 & 4 & 0 \end{pmatrix}$$

2) Which of the following sets S are subspaces of R^2 ? Explain each answer briefly.

a) $S = \{(x_1, x_2)^T |\ 3x_1 + 4x_2 = 7\}$

b) $S = \{(x_1, x_2)^T |\ x_1 = x_2\}$

c) $S = \{(x_1, x_2)^T |\ x_1 = x_1\}$

d) $S = \{(x_1, x_2)^T |\ x_1^2 + x_2^2 = 0\}$

3) Choose ONE of these to prove. You can answer on the back.

a) If dim(V)$= n > 0$ and $B = \{v_1, v_2, \dots, v_n\}$ spans V then B is a basis of V.

b) Show that a nonempty subset of a linearly independent set of vectors $\{v_1, v_2, \dots v_n\}$ is also linearly independent.

c) Let $L = \{v_1, v_2, \dots, v_n\}$ be a spanning set of V and let $v \in V$ be any other vector. Show that $\{v_1, v_2, \dots, v_n, v\}$ is linearly dependent.

Extra Credit (about 3-5 points): Should a Linear Algebra course include MATLAB in the lectures and homework? Why, or why not?

Answers: The average grade was about 35/60.

1) Find the solution set to $Ax = 0$ from U. After factoring out the α and β, you should get $N(A) = \text{span}\ \{[0\ \ -3\ \ -4\ 1\ 0]^T, [-1\ \ -1\ 0\ 0\ 1]^T\}$. Since these two are not scalar multiples of each other they are LI and are a basis.

2) I accepted various explanations. For example, it is OK to check the closure properties. But here are the simplest answers:

2a) No. S does not contain the zero vector.
2b) Yes. $S = \text{span}\ \{[1\ 1]^T\}$.
2c) Yes. $S = R^2$.
2d) Yes. $S = \{\mathbf{0}\}$.

3) a) is part of the "2/3 theorem" and is in the book. The other two were HW:

3b) We can assume the subset is the shortened list, $\{v_1, v_2, \dots v_j\}$ where $1 \le j \le n$. We can assume that $c_1 v_1 + \dots c_j v_j = 0$ and must prove the c's are all zero. But by adding in zeroes, we can change this equation to $c_1v_1 + \dots c_jv_j + 0v_{j+1} + \dots 0v_n = 0$. Since the original list is LI, all the c's have to be zero. Done.

3c) Since $v \in$ span (L), we get $c_1v_1 + \dots c_nv_n = v$. And by subtracting, $c_1v_1 + \dots c_nv_n - v = 0$. The coefficient of v is not zero, so this shows LD.

The answers to the bonus question were mostly positive, especially about including MATLAB in the lectures (about 7-1). Most people also think it should be part of the HW (about 5-3) but several people consider it too time-consuming. Compared to other semesters, there were almost no complaints about not understanding the exercises or the MATLAB commands.