

Show all your work and reasoning for maximum credit. If you continue your work on another page, be sure to leave a note. Do not use a calculator, book, or any personal paper. You may ask about any ambiguous questions or for extra paper. If you use extra paper, hand it in with your exam.

- 1) The matrix A is row equivalent to U . a) Find a basis for $N(A)$, and explain briefly how you know it is a basis. b) Do the same for $\text{Col } A$.

$$A = \begin{pmatrix} 1 & 3 & 0 & 9 & 4 \\ 1 & 1 & 4 & 19 & 2 \\ 1 & 1 & 2 & 11 & 2 \end{pmatrix} \quad U = \begin{pmatrix} 1 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 3 & 1 \\ 0 & 0 & 1 & 4 & 0 \end{pmatrix}$$

- 2) Are the following sets subspaces of P_4 ? Be careful! And explain each answer briefly.

a) The polynomials $p(x)$ in P_4 such that $p(0) = 0$.

b) The polynomials in P_4 of even degree.

- 3) Choose ONE of these to prove. You can answer on the back.

a) If $\dim(V) = n > 0$ and $B = \{v_1, v_2, \dots, v_n\}$ is L.I., then B is a basis of V .

b) If U and V are subspaces of W then so is $U \cap V$.

c) If $B = \{v_1, v_2, \dots, v_n\} \subset V$, then $\text{span}(B)$ is a subspace of V .

Answers: The average grade this time was barely over 30/60, with fairly poor average results on all three problems.

1a) See Spring 2004, Quiz 4.

1b) The leading ones in U appear in columns 1, 2 and 3. So, the first three columns of A are a basis for $\text{Col}(A)$.

2a) Yes. It is closed under addition; if p and q are in the set S , then $p(0) = 0$ and $q(0) = 0$. So, $(p + q)(0) = p(0) + q(0) = 0$ and $p + q \in S$. A similar calculation shows S is closed under scalar multiplication.

2b) No. For example, $x^2 + x \in S$ and $-x^2 + 1 \in S$, but when we add these, we get $x + 1 \notin S$.

This problem was from Ch 3.2-5.

3) Parts a) and c) are in the book and the lectures. Part b) is from the exercises.