MAS 3105 — March 1, 2007

Quiz 4 and Key — Prof. S. Hudson

1) Use a Wronskian to determine whether the three vectors 1, e^x, e^{-x} are LI in $C[0,1]$:

2) Find the transition matrix representing the change in coordinates from the basis $[x, 1]$ of P_2 to the basis $[3x-1, 3x+1]$.

3) Choose ONE of these to prove. You can answer on the back.

a) If dim(V)= $n > 0$ and $B = \{v_1, v_2, \ldots, v_n\}$ is L.I., then B is a basis of V.

b) Two vectors in V are L.D. if and only if one is a scalar multiple of the other.

c) Suppose A is 4x4 and is row equivalent to B. Suppose the first three columns of A are L.D. Then the first three columns of B are also L.D.

Answers and Remarks: The average was about 40/60. A's = 48-60, B's = 42-47, etc.

1)

$$W(x) = \det \begin{pmatrix} 1 & e^x & e^{-x} \\ 0 & e^x & -e^{-x} \\ 0 & e^x & e^{-x} \end{pmatrix} = 2$$

since $e^x e^{-x} = 1$. Since $2 \neq 0$ they are LI.

2)

$$\begin{pmatrix} 3 & 3 \\ -1 & 1 \end{pmatrix}^{-1} = \begin{pmatrix} 1/6 & -1/2 \\ 1/6 & 1/2 \end{pmatrix}$$

3a) See the proof of the 2/3's theorem (show the vectors span V).
3b) was HW
3c) Basically, repeat our discussion of dependency relations; that the columns of A and B have the same ones.