MAS 3105 — Oct 17, 2008

Quiz 4 and Key — Prof. S. Hudson

1) The matrix A is row equivalent to U. a) Find a basis for $N(A)$, and explain briefly how you know it is a basis. b) Do the same for Col A.

$$A = \begin{pmatrix} 1 & 2 & -1 & 1 \\ 2 & 4 & -3 & 0 \\ 1 & 2 & 1 & 5 \end{pmatrix} \qquad U = \begin{pmatrix} 1 & 2 & 0 & 3 \\ 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

2) Are the following sets subspaces of R^3 ? Explain each answer briefly.

a) $S = \{(x_1, x_2, x_3)^T \mid x_1 = x_2 \text{ and } x_3 = 0\}$

b) $S = \{(x_1, x_2, x_3)^T \mid x_1 = x_2 \text{ or } x_1 = x_3\}$

3) Choose ONE of these to prove. You can answer on the back.

a) If dim(V)$= n > 0$ and $B = \{v_1, v_2, \ldots, v_n\}$ is L.I., then B is a basis of V.

b) If U and V are subspaces of W then so is $U \cap V$.

c) If $B = \{v_1, v_2, \ldots, v_n\} \subset V$ is L.I., then any subset of B is also L.I.

Remarks and Answers: I don't have the scores from the grader yet, but he estimates that the average was about 35/60. If so, an approx scale is A's 45-60, B's 39-44, C's 33-38, D's 27-32.

He said some students couldn't write proofs, using the definition of LI for example. You had graded HW on that, so I suppose you know whether this is a problem for you. If you aren't comfortable with proofs yet, come by my office for help (or visit Rafael, or read Velleman, etc)

1a) $N(A) = N(U)$ is spanned by $\{[-3\ 0\ -2\ 1]^T, [-2\ 1\ 0\ 0]^T\}$ based on the usual calculations. Since the two vectors are not scalar multiples of each other, they are LI and form a basis.

1b) Cols 1 and 3: $\{[1\ 2\ 1]^T,\ [-1\ -3\ 1]^T\}$ (calcs and explanation similar to exs from class and text)

2a) Yes. There are many ways to check this, such as one of these formulas: $S = N(A)$ (and find A), or $S = \text{span}\ \{[1\ 1\ 0]^T\}$.

2b) No. I don't think there is any simple way to explain this without giving a counterexample to closure under addition. For example, $[1\ 1\ 0]^T + [1\ 0\ 1]^T = [2\ 1\ 1]^T \notin S$. You could say that S is the union of two planes in R^3, but that is not really a complete explanation.

3a) This is part of the 2/3 thm, done in class and the text.

3b) This is HW 3.2.18. Use the def of subspace and this should be pretty easy.

3c) This is HW 3.3.13. Assume B is LI and that $L \subset B$. We can assume that L consists of the first k vectors in B, $k < n$ [It takes some thought, or some experience, to decide what it is OK to assume in a proof. In this case, the order of the vectors in B clearly doesn't matter, so we can simplify our proof a little by making this assumption]. Now assume

$$c_1\mathbf{v_1} + \ldots c_k\mathbf{v_k} = \mathbf{0}$$

[We can assume this because it is the first part of the definition of 'L is LI'. We must show these c_i are all zero]. We can add zeroes into this formula to get

$$c_1\mathbf{v_1} + \ldots c_k\mathbf{v_k} + 0\mathbf{v_{k+1}} + \ldots + 0\mathbf{v_n} = \mathbf{0}$$

Now, since we know B is LI, all the coefficients in this formula must be zeroes. So, all the ones in the previous formula are zeroes and L is LI.

Rk: Suppose you begin the proof of 3c) with the definition of LI for B instead,

$$c_1\mathbf{v_1} + \ldots c_n\mathbf{v_n} = \mathbf{0}$$

This breaks no rules, but I don't see any clear way to continue the proof. You can say that all these $c_i = 0$, but you haven't proved anything about L (these c_i may not be the same ones appearing in the definition of 'L is LI' - it all depends on how you introduce the c_i into your proof].