MAS 3105 | AM: Feb 28, 2008
Quiz 4 and Key | Prof. S. Hudson

1) [40 pts] Set $B = \{\mathbf{v_1}, \mathbf{v_2}, \mathbf{v_3}\} = \{(1,0,0)^T, (1,1,0)^T, (1,0,1)^T\}$. So, B is a set of 3 column vectors in R^3. Answer, and explain briefly:

a) Is B a basis of R^3?

b) Find the transition matrix from B to the standard basis of R^3.

c) Notice that $\mathbf{w} = (3,1,0)^T$ is a simple LC of $\mathbf{v_1}$ and $\mathbf{v_2}$. Find the coordinates of $\mathbf{w}$ with respect to B. Write your answer as a column vector.

d) Suppose $B_2 = \{\mathbf{v_3}, \mathbf{v_2}, \mathbf{v_1}\}$. Find the transition matrix from B_2 to B.

e) Find the coordinates of $\mathbf{w}$ with respect to B_2 (as in part c). Check your answer to (d) using your answers to (c) and (e).

2) Choose ONE of these to prove. You can answer on the back.

a) If the columns of A are LI and $Ax = b$ is consistent, then the system has a *unique* solution. This is a rephrasing of Thm 3.3.2; so, prove it, don't just quote it.

b) If dim(V)$= n > 0$ and $B = \{v_1, v_2, \ldots, v_n\}$ spans V, then B is a basis of V.

c) Any two bases of V must have the same number of elements. [This is a corollary of Thm 3.4.1, which you can quote without proof - IF you remember it!]

Remarks and Answers: The average on this quiz was about 43/60, which is fairly normal for a Quiz 4. The approx scale is A's 48-60, B's 42-47, C's 36-41, D's 30-35.

I wrote your semester average in the upper right corner of your quiz, based on your 4 quiz scores (still including your lowest score, but not your HW or MHW). The class average for this is about 46. The approx scale is: A's = 50 to 60, B's = 44 to 49, C's = 38 to 43, D's = 32 to 37. If you have a very low score (more than 15 points below your average) and are doing OK on the MHW, then your letter grade may go up a bit later. Otherwise, it may go down a bit, since the scale will probably rise a little then. Drops may have a small effect, too, but of course, your future grades, and HW, will probably have a greater effect.

1a) Yes. Since the matrix (see 1b)) is nonsingular, the columns are LI and span R^3.

1b) This is the 'easy direction':

$$U = \begin{pmatrix} 1 & 1 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

1c) $\mathbf{w} = 2\mathbf{v_1} + 1\mathbf{v_2}$, so $\mathbf{w} = [2\ 1\ 0]^T$, in B-coordinates. You can also compute this from $U^{-1}[3\ 1\ 0]^T$, since U^{-1} converts from standard to B. But a common mistake was to try $U[3\ 1\ 0]^T$ instead.

1d) The transition matrix will interchange the 1st and 3rd coordinates of vectors like $\mathbf{w}$. So, it is elementary of type I (you can also compute it from $V = U^{-1}U_2$, where U_2 is obtained as in 1b), but from B_2):

$$V = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{pmatrix}$$

1e) The first step is very similar to 1c), but use B_2.

$$\mathbf{w} = \begin{pmatrix} 2 \\ 1 \\ 0 \end{pmatrix}_B = \begin{pmatrix} 0 \\ 1 \\ 2 \end{pmatrix}_{B_2} \quad \text{and} \quad \begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{pmatrix} \begin{pmatrix} 0 \\ 1 \\ 2 \end{pmatrix} = \begin{pmatrix} 2 \\ 1 \\ 0 \end{pmatrix}$$

2) See text/lecture for model answers. Remarks:

a) Given this phrasing, you should use $\mathbf{a_j}$ instead of $\mathbf{v_j}$, and use $\mathbf{b}$ instead of $\mathbf{v}$. Also, this is only half of Theorem 3.3.2, so don't prove both directions (I don't usually take points off for that, but it wastes your quiz time, and if the overall confusion level gets too high, I do take off points).

b) This is really just one part of the 'two-thirds theorem', which everybody should have learned before the quiz. But most answers didn't include the main idea (removing vectors from B), and a few merely quoted the two-thirds theorem (not allowed!). I gave full credit to anyone who even mentioned the main idea.

c) Most people who tried this got the main idea right, which is pretty simple, that the larger set must be LD. But for full credit, you had to quote Theorem 3.4.1 fairly accurately. For example, saying that the smaller set cannot span V is a valid consequence of the theorem, but it is not a direct quote, and it is not necessary for this proof.

Also, several people assumed that the larger set had exactly *one* more vector than the other; a minor error of logic.