MAS 3105 PM: Feb 28, 2008

Quiz 4 and Key Prof. S. Hudson

1) Set $B = \{\mathbf{v_1}, \mathbf{v_2}, \mathbf{v_3}, \mathbf{v_4}\} = \{(1,0,0)^T, (1,1,0)^T, (1,0,1)^T, (1,1,1)^T\}$. So, B is a set of 4 column vectors in R^3. Answer, and explain briefly:

a) Is B a spanning set of R^3?

b) Is B linearly independent?

c) Is B a basis of R^3?

d) Is Span$\{\mathbf{v_1}, \mathbf{v_3}\}$ a subspace of R^3 ?

e) Let $C = \{\mathbf{v_1}, \mathbf{v_2}, \mathbf{v_3}\}$, which is a basis of R^3. Find the transition matrix U from this basis to the standard basis of R^3.

2) Answer True or False:

The columns of a nonsingular matrix are always linearly independent (LI).

If L is a list of n LI vectors, then dim (span L) $= n$.

If A is a singular 4x4 matrix, then its rank is at most 3.

A transition matrix from one basis to another always has a nonzero determinant.

If L is a list of 3 LI functions in P_3, then span $(L) = P_3$.

3) Choose ONE of these to prove. You can answer on the back.

a) If dim(V)$= n > 0$ and $B = \{v_1, v_2, \ldots, v_n\}$ is LI, then B is a basis of V.

b) If dim(V)$= n > 0$ and $B = \{v_1, v_2, \ldots, v_n\}$ spans V, then B is a basis of V.

c) Any nonempty subset of an LI set is also LI.

Remarks and Answers: The average on this quiz was about 44/60, which is fairly normal for a Quiz 4. The approx scale is A's 49-60, B's 43-48, C's 37-42, D's 31-36.

I wrote your semester average in the upper right corner of your quiz, based on your 4 quiz scores (still including your lowest score, but not your HW or MHW). The class average for this is about 45, so it has about the same scale as for Quiz 4 above. If you have one very low quiz score (more than 15 points below your average) and are doing OK on the MHW, then your letter grade may go up a bit later. Otherwise, it may go down a bit, since the scale will probably rise a little then. Drops may have a small effect, too, but of course, your future grades, and HW, will probably have a greater effect.

1a) Yes. The first three vectors form a nonsingular matrix, so even those three span R^3 (see 1e)).

1b) No. It has more vectors than dim R^3, so the set is LD.

1c) No. It's LD.

1d) Yes. A theorem of Ch 3 says that span(L) is a subspace.

1e) This is the easy direction (no need to take U^{-1}):

$$U = \begin{pmatrix} 1 & 1 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

2) All are True (which happened randomly).

3) See tex/lectures. I think (c) is the easiest if you didn't prepare carefully for (a) and (b) [but you should've]. Some common mistakes were:

- Choosing part a) but giving a proof of b) [for example]. If the proof is clear enough, this is not a big problem. But overall clarity is part of the grade for proofs.

- a) and b) are slightly rephrased parts of the proof of the 2/3 theorem, which I asked everybody to learn for the quiz. So, you cannot simply quote that theorem and call it a proof.

- Some answers were rambling essays, containing mostly-true statements, which were not well-justified. You should justify each statement (unless it follows logically from the previous line(s)) with a definition, or a previous theorem. You don't have to know the theorem number, but should usually state precisely what it says.

Sometimes you must make judgement calls about what to explain. But I usually give you a short list of proofs to study, and you can normally sort this out before the quiz. You can come by my office about that, and you can also ask me about specific issues during a quiz. But if you simply point to your proof and ask 'Is this enough?', I probably won't look very closely; I can't help you find gaps and errors during the quiz.

- Some people wrote nonsense phrases like "... so V is a LC of S ... (a vector can be a LC, but a vector space can't). In these cases, the rest of the proof was usually wrong, too, so perhaps the basic mistake was not studying enough.

You might also look at my answer key to the AM Quiz 4, and/or answer keys from previous semesters.