MAS 3105 May 28, 2009

Quiz 4 and Key Prof. S. Hudson

1) Let $\mathbf{x_1} = (1,1,1)^T$ and $\mathbf{x_2} = (3,-1,4)^T$. Find a third vector $\mathbf{x_3}$ so that $\{\mathbf{x_1}, \mathbf{x_2}, \mathbf{x_3}\}$ is a basis of R^3. Explain briefly how you know this is a basis.

2) Which of the following are subspaces of R^3? Explain each answer briefly.

a) $S = \{(x_1, x_2, x_3)^T |\ 3x_1 + x_3 = 0\}$

b) $S = \{(x_1, x_2, x_3)^T |\ x_1 x_3 = x_2\}$

c) $S = \{(x_1, x_2, x_3)^T |\ x_1 = 5x_2 = x_3\}$

d) $S = \{(x_1, x_2, x_3)^T |\ \ |x_1| + |x_2| = 1\}$

3) Choose ONE

a) Prove this part of the 2/3 thm: If L is a L.I. set of n vectors in V, and dim$V = n$, then L spans V.

b) Let L be a finite list of vectors in V. Prove that span (L) is a subspace of V. To save time, you can skip the parts about $L \subset V$ and about closure under addition. But prove the other two parts.

c) Suppose A is a nonsingular 3x3 matrix. Prove that its columns span R^3. Include the definition of span (don't take a shortcut such as saying that the columns are actually a basis).

Remarks and Answers: The average was about 40 out of 60, the lowest one so far, but not very unusual for a Ch 3 quiz. As I recall the results were good on problem 1, normal on 2, and a bit low on 3.

Your semester grade,so far: This is tricky, since your lowest quiz score will be dropped, but we don't know if you've already gotten your lowest one or not. Here are two reasonable methods to compute your semester grade at this point:

1) You can average out your 4 quiz grades (ex: B+, B, B+, C+ averages to a B) based on the 4 scales given in the answer keys.

2) I wrote my estimate in the upper right corner of your Quiz 4. I added your best three quiz scores, and placed that on a new scale with A's = 156 - 180, B's = 144 to 155, C's 130 to 143. So far, I have not included HW or MHW into your grade.

1) You can choose $\mathbf{x_3} = (1,0,0)^T$ (or almost any randomly chosen vector). Since det $[\mathbf{x_1}, \mathbf{x_2}, \mathbf{x_3}] \neq 0$, these vectors are a basis of R^3. Since finding $\mathbf{x_3}$ was so easy, the expla-

nation counted for a lot of the grade, with no partial credit for bad choices of $\mathbf{x_3}$.

2a) Yes. I accepted most explanations that used words. Any one of these is OK:

* S is closed under addition and scalar mult. It is not quite obvious that it is closed, but I gave credit for this anyway. This was the most common type of answer.

* It is a plane through the origin.

* $S = \text{span}\ ([0,1,0]^T, [1,0,-3]^T)$

* $S = N([3,0,1])$. This is probably the simplest explanation, if you see it.

2b) No. It fails closure under scalar mult (giving a counterexample is even better).

2c) Yes. It is a line through the origin. It is spanned by $[5,1,5]^T$.

2d) No. It does not contain the 0 vector.

3) See the text or lectures. For 3b), the instructions imply you must prove that span (L) is nonempty (even if this is not done in the book). It is fairly easy, just mention that the zero vector equals the trivial LC, and every LC is in the span. For 3c), explain that $Ax = b$ is always consistent (etc).