MAS 3105 March 13, 2003
Quiz 5 Prof. S. Hudson

1) Answer TRUE or FALSE:

a) Two row equivalent matrices must have the same rank and the same nullity.
b) Two row equivalent matrices must have the same column space.
c) If A is similar to a singular matrix then A is also singular.
d) Ker (L) is a subspace of V that depends on which basis is used for V.
e) If A represents $L : R^3 \to R^2$ then A is a 2x3 matrix.

2) Find the transition matrix for the change in coordinates from the basis $[x, 1]$ of P_2 to the basis $[2x - 1, 2x + 1]$.

3) Choose ONE of these proofs.

a) If A and B are 3x3 matrices then rank $(AB) \leq$ rank (A).

b) If Ker $(L) = \{\mathbf{0}\}$ then L is 1-1.

c) Thm 3.6.6: Dim (Row (A)) = Dim (Col(A)).

Answers: 1) TFTFT

2) This HW 3.5 9b (which resembles Ex.7 on page 172). The first basis can be treated as the "standard" one, because it is easy to find the coordinates of any vector in P_2 using it. If U goes from the second to the first, the first column of U is $2x - 1$ written as a column vector, $[2, -1]^T$ (this matrix U answers HW 9a). But the answer to 2) [and to 3.5 9b] is U^{-1}. Use GE to find it. *Suggestion: always check your answer to such inverse calculations.*

$$U = \begin{pmatrix} 2 & 2 \\ -1 & 1 \end{pmatrix} \quad \text{so,} \quad U^{-1} = \begin{pmatrix} 1/4 & -1/2 \\ 1/4 & 1/2 \end{pmatrix} \quad \text{and} \quad UU^{-1} = I$$

3a) This is part of HW 3.6 19c. Explain that each column of AB is a LC of the columns of A so that Col $(AB) \subseteq$ Col (A). So, dim Col $(AB) \leq$ dim Col A, which implies that rank $AB \leq$ rank A.

3b) This is HW 4.1 20 and was done in class.

3c) Thm 3.6.6. See the proof in the text. Or the one from the lectures, but be sure to explain each step:

dim col A	= dim col U	b/c same dependency relations
	= number of leading ones in U	since columns with ones form a basis
	= number of nonzero rows in U	there's a 1-1 correspondence
	= dim row U	since the nonzero rows are LI
	= dim row A	since row A = row U

The first step should probably be explained more fully (see lecture notes or page 178 and Ex4 on 179).