MAS 3105 June 9, 2005
Quiz 5 and Key Prof. S. Hudson

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**Name**

Show all your work and reasoning for maximum credit. If you continue your work on another page, be sure to leave a note. Do not use a calculator, book, or any personal paper. You may ask about any ambiguous questions or for extra paper. If you use extra paper, hand it in with your exam.

1) [20 pts] Answer TRUE or FALSE:

Two row equivalent matrices must have the same column space.

If $A$ is similar to $A^2$, then $A^2$ is similar to $A^4$.

If $A$ represents $L : R^3 \to R^2$ then $A$ is a 2x3 matrix.

If $A$ is an mxn matrix then $A$ and $A^T$ have the same rank.

If $A$ is an mxn matrix then $A$ and $A^T$ have the same nullity.

2) [20 pts] Let $L : R^2 \to R^2$ be a linear transformation such that $L((1,2)^T) = (2,3)^T$ and $L((2,3)^T) = (4,5)^T$. Find $L((3,4)^T)$.

3) [20 pts] Choose ONE of these to prove.

a) *Edited 6/13/05*: If $L : V \to W$ is linear, and $T$ is a subspace of $W$, then $L^{-1}(T)$ is a subspace of $V$ .

b) Prove that $\dim(\text{Row}(A)) = \dim(\text{Col}(A))$. [If you follow my outline, explain each step and include the phrase "dependency relation" when it is needed].

c) Show that if $A$ is similar to $B$ then $A^T$ is similar to $B^T$.

**Answers:**

1) FTTTF

2) $[6\ 7]^T$. I used $[3\ 4]^T = 2[2\ 3]^T - [1\ 2]^T$ to get this, but there are other ways (you could find the matrix rep for $L$ instead, for example).

3) a) = HW and b) = Thm. The easiest is part c): Since $A = S^{-1}BS$, we can transpose both sides and get $A^T = S^T B^T (S^{-1})^T$. But $(S^{-1})^T = (S^T)^{-1}$, by previous HW, so $A^T$ is similar to $B^T$.