MAS 3105 Nov 8, 2006
Quiz 5 Key Prof. S. Hudson

1) Answer TRUE or FALSE: As usual, R^2_E means column vectors in R^2, written using the basis E.

If $L : R^1 \to R^1$ is linear, then $L(x) = L(1)x$, for all $x \in R^1$.
The transformation $L(A) = A^T$ is linear, from R^{3x2} to R^{2x3}.
If A is similar to I, then $A = I$ (I is the identity matrix).
If A represents $L : R^2_E \to R^2_E$ and B represents $L : R^2_F \to R^2_F$, then A is similar to B.
If A represents $L : R^2_E \to R^2_F$ and B represents $L : R^2_F \to R^2_E$, then A is similar to B.

2) Find the matrix representation of $L : R^2 \to R^2$, where L is a 90 degree clockwise rotation [use the standard basis of R^2].

3) Choose ONE of these. You can answer on the back.

a) Prove thm 5.1.1; that in R^2, $\mathbf{x}^T\mathbf{y} = ||\mathbf{x}||\ ||\mathbf{y}||\ \cos(\theta)$. Include a picture.

b) HW 4.2.20: Assuming that $L : V \to W$ is linear and T is a subspace W, prove that $L^{-1}(T)$ is a subspace of V. Include the definition of $L^{-1}(T)$ in your answer (this will also justify some partial credit).

Remarks and Answers: The average was about 40/60. A's = 48 to 60, and subtract about 6 points per letter. I wrote your estimated semester grade in the corner, based on your best 4 out of 5 quiz scores, so far.

1) TTTTF

2) Similar to Ch 4.2.5. Compute $L(e_1) = [0,\ -1]^T$ to get the first column, etc:

$$A = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$$

3a) See the textbook. For full credit, explain every step. Mention the Law of Cosines by name, and that $||x||^2 = x^T x$ and $x^T y = y^T x$.