MAS 3105 March 15, 2007

Quiz 5 Key Prof. S. Hudson

1) Find the matrix representation $A$ of the linear operator $L : R^2 \to R^2$ that reflects each vector $\mathbf{x}$ about the line $x_2 = x_1$ and then projects it onto the $x_1$-axis.

2) Answer TRUE or FALSE: Part a) refers to the 'baby rabbit example' from the lectures.

a) With 2 adults and 3 baby rabbits this year, we'll have 9 rabbits total, next year.

b) The commuting diagram for Ch 4.3 (similarity) contains only one vector space, $V$.

c) If $A$ is similar to itself, then $A$ is nonsingular.

d) If $A$ and $B$ are row equivalent they have the same column space.

e) The matrix $A$ that answers problem 1) has rank = 1.

3) Choose ONE of these.

a) Suppose $A$ and $B$ are nxn and $B$ is nonsingular. Prove that rank $(BA)$ = rank $(A)$. [Hint: compare their nullspaces]

b) Suppose that $L : V \to W$ is linear and $S$ is a subspace of $V$. Prove that $L(S)$ is a subspace of $W$.

c) State the formula in Thm 5.1.1 for $\mathbf{x}^\mathbf{T}\mathbf{y}$ and prove it (in $R^2$).

**Remarks and Answers:** The average was about 40/60. The unofficial scale is: A's 45-60, with 6 points per letter below that. I computed your semester average, based on your best 4 out 5 quiz scores so far, and wrote a corresponding letter grade in the upper corner of your quiz. The scale for that was: A's 200-240, B's 175-199, etc (25 points per letter). I will include HW and MHW later, of course.

1) $a_1 = L(e_1) = (0, 0)^T$ and $a_2 = L(e_2) = (1, 0)^T$ (combine to get $A$).

2) TTFFT

3a) Expected proof (using the hint): If $Ax = 0$ then $BAx = 0$. The converse is also true, using $B^{-1}$. So, $N(BA) = N(A)$ and the two matrices have the same nullity. Also, they are both nxn. By the rank-nullity theorem, they have the same rank, $n-$ dim $N(A)$.

Alternative proof: From previous chapters, we know that $B$ can be factored into elementary matrices, so $BA$ is row equivalent to $A$. By Thm 3.6.1, Row $(BA)$ = Row $(A)$. Now take dim of both sides.

b) Nobody got this one, maybe because you need two definitions (of subspace and $L(S)$ ). Recall $L(S) = \{L(s) : s \in S\}$.

It is easy to see that $L(S) \subset W$. Also, since $0 \in S$, we know $0 = L(0) \in L(S)$, so it is not empty. Now for closure; suppose $v_1$ and $v_2$ are vectors in $L(S)$. This means $\exists s_1, s_2 \in S$ such that $L(s_1) = v_1$ and $L(s_2) = v_2$. So, $L(s_1 + s_2) = L(s_1) + L(s_2) = v_1 + v_2$. Since $S$ is a subspace of $V$, we know $s_1 + s_2 \in S$, so the previous sentence shows $v_1 + v_2 \in L(S)$. I will leave scalar mult to you.