

1) Suppose that  $L((1, 2)^T) = (5, 11)^T$  and  $L((3, 4)^T) = (11, 25)^T$ . Find  $L((4, 6)^T)$ .

2) Find the transition matrix from  $[\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3]$  to  $[\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3]$  where:

$$\mathbf{v}_1 = (4, 6, 7)^T, \mathbf{v}_2 = (0, 1, 1)^T, \mathbf{v}_3 = (0, 1, 2)^T$$

$$\mathbf{u}_1 = (1, 1, 1)^T, \mathbf{u}_2 = (1, 2, 2)^T, \mathbf{u}_3 = (2, 3, 4)^T$$

3) Choose ONE of these to prove (on the back of the page).

- If  $A$  is similar to  $B$  then  $A^T$  is similar to  $B^T$ .
- Suppose that  $L : V \rightarrow W$  is linear. Prove that  $\text{Ker}(L)$  is a subspace of  $W$ .
- The dimension of  $\text{Col}(A)$  equals the dimension of  $\text{Row}(A)$ .

Bonus (about 5 pt): Give an example of a linear transformation which does not have a matrix representation.

**Remarks and Answers:** The average grade was about 50/60. You can use this (unofficial) scale: A's = 54 to 60, B's = 48 to 53, C's = 43 to 38, D's = 33 to 37.

You can estimate your semester grade using the same scale. Average your best 4 of 5 quiz grades. Compare that number with the scale above. The class average for this number is also about 50/60. This is a bit inflated, because I just dropped a grade and because the scores were fairly high on Q5; the scale will probably come down a little with Q6 and the final. Of course, I will also include your HW and MHW later on.

1) Notice that  $(4, 6)^T = (1, 2)^T + (3, 4)^T$  (this made the problem fairly easy - otherwise you'd have to solve a linear system (or compute a transition matrix) to find the right LC). So,  $L((4, 6)^T) = L((1, 2)^T) + L((3, 4)^T) = (16, 36)^T$ .

2)

$$VU^{-1} = \begin{pmatrix} 1 & -1 & 2 \\ 1 & 1 & 0 \\ 1 & 0 & 1 \end{pmatrix}$$

3) Parts b) and c) are in the text/lectures, but part a) is easiest.

B) Any example of  $L : V \rightarrow W$  in which  $V$  (or  $W$ ) is infinite-dimensional. In that case, the  $v \in V$  can't be represented as column vectors. One example is  $d/dx : P \rightarrow P$ .