MAS 3105 AM: March 13, 2008

Quiz 5 Prof. S. Hudson

1) Answer TRUE or FALSE: [the '$A$' in part b) is *not* the same as in c), and so on]

a) Two row equivalent matrices must have the same rank and the same nullity.
b) $A\mathbf{x} = \mathbf{b}$ is consistent if and only if $\mathbf{b}$ is in the column space of $A$.
c) If $A$ is a transition matrix, then $A$ is square and det $A \neq 0$
d) The rank of $A$ is greater than or equal to the number of columns of $A$.
e) If $A$ represents $L : R^3 \to R^2$ then $A$ is a 2x3 matrix.

2) These two matrices are row equivalent:

$$A = \begin{bmatrix} 1 & 2 & 0 & 3 & 0 \\ 1 & 2 & 1 & 3 & 1 \\ 2 & 4 & 0 & 6 & 7 \end{bmatrix} \text{ and } U = \begin{bmatrix} 1 & 2 & 0 & 3 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}.$$

a) Find a basis of the row space of $A$.

b) Find a basis for the column space of $A$.

c) Find a basis for the nullspace of $A$.

d) Find the rank of $A$.

e) Find the nullity of $A$.

3) Choose ONE of these.

A) Let $L$ be the operator on $P_3$ defined by $L(p(x)) = xp''(x) + p(1)$. Find the matrix $A$ representing $L$ with respect to $[1, x, x^2]$.

B) State and prove Theorem 5.1.1, the formula for the dot product which includes $\theta$.

C) Let $S$ be a subspace of $R^3$ and let $L : R^3 \to R^3$ be linear. Show that $L(S)$ is a subspace of $W$ [so, prove part of thm. 4.1.1].

**Remarks and Answers:** The average for Quiz 5 was about 51 (good!). The unofficial scale is: A's 54-60, B's 49-53, C's 43-48, D's 37-42.

I have computed the average of your best 4 out of 5 quiz grades. Assuming that you are doing OK on the HW and the Matlab HW, this is probably a good estimate of how you are doing so far (but a bit high). The class average for this score is about 51/60. Since this is about the same as for Quiz 5, use the same scale (see above). Also, see the upper right corner of your Quiz 5. Answers:

1) TTTFT

2a) The standard answer is: the three nonzero rows of $U$. You can also use the three rows of $A$, *in this example*, but I'd like to see some reasoning.

2b) Use columns 1, 3 and 5 of $A$. Again, other less standard answers are possible, but I'd like to see some reasoning.

2c) $\{[-2\ 1\ 0\ 0\ 0]^T,\ [-3\ 0\ 0\ 1\ 0]^T\}$. As usual, I used the transpose symbol to type these column vectors more easily. You don't need to do that if you are using a pencil (but I didn't mark off for it). Some people took this to the extreme and wrote the row vectors in 3a as transposed column vectors. There is no reason for that!

2d) 3

2e) 2

The results were pretty good on 3a) and b), but a bit lower on c), though it was an advertised proof (Thm 4.1.1). I'll try to walk you through part of that proof, in a little more detail than the textbook.

3c) Suppose that $L : V \to W$ is linear and $S$ is a subspace of $V$. Prove that $L(S)$ is a subspace of $W$.

Partial Proof: Referring to the definition of *subspace*, we must mainly prove the two closure properties. I'll do one here (closure under addition). Assume that $\mathbf{w_1}$ and $\mathbf{w_2}$ are in $L(S)$ (since $L(S) \subset W$, it makes sense to label these with $w$'s) rather than $v$'s). We must show that $\mathbf{w_1} + \mathbf{w_2}$ is also in $L(S)$. Now by definition, $\mathbf{w_1} \in L(S)$ means there is a vector $\mathbf{v_1} \in S$ such that $L(\mathbf{v_1}) = \mathbf{w_1}$ (this definition is what most people missed). Likewise, we get $L(\mathbf{v_2}) = \mathbf{w_2}$. Then by linearity, $L(\mathbf{v_1} + \mathbf{v_2}) = L(\mathbf{v_1}) + L(\mathbf{v_2}) = \mathbf{w_1} + \mathbf{w_2}$. Since $S$ is a subspace, $\mathbf{v_1} + \mathbf{v_2} \in S$, so this calculation shows $\mathbf{w_1} + \mathbf{w_2} \in L(S)$ (using the def again). I'm done (but you should also prove closure under scalar mult, and nonempty, as in the text).