

1) [30pts] These two matrices are row equivalent. U is in REF but not RREF.

$$A = \begin{pmatrix} 5 & 0 & 5 & 0 \\ 3 & 2 & 5 & 6 \\ 2 & 0 & 2 & 3 \end{pmatrix} \quad U = \begin{pmatrix} 1 & 0 & 1 & 7 \\ 0 & 1 & 1 & 3 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

a) Find a dependency relation for the columns of A .

b) Find a basis of $\text{col}(A)$.

c) Find a basis of $N(A)$ and find the nullity of A .

2) [10pts] Find the transition matrix from the standard basis of R^2 to the basis $B = \{[0, 1]^T, [2, 0]^T\}$. Circle your answer.

3) Circle ONE of these proofs, and answer on the back.

a) Let $L : V \rightarrow W$ be linear. Prove that $\text{Ker}(L)$ is a subspace of V .

b) If $\text{Ker}(L) = \{\mathbf{0}\}$ then L is 1-1.

c) Thm 3.6.6: $\text{Dim}(\text{Row}(A)) = \text{Dim}(\text{Col}(A))$.

Remarks and Answers: The average was approx 47 out of 60, which is pretty good, especially for Chapter 3 material. Many people didn't seem to know exactly what a dependency relation is (1a), but the results on the other problems were good. The scale is A's = 52-60, B's = 46-51, C's = 40-45, etc.

My new estimate for your semester grade is in the upper right. It is based on your best four quiz grades, so far. The average for that is approx 194 out of 240.

1a) $\mathbf{a}_1 + \mathbf{a}_2 - \mathbf{a}_3 + 0\mathbf{a}_4 = \mathbf{0}$. A dependency relation is an equation that shows a set of vectors fits the definition of LD. Ideally, it should contain all 4 vectors and should have the zero vector on the RHS.

1b) Use the leading ones in U to choose $\{\mathbf{a}_1, \mathbf{a}_2, \mathbf{a}_4\}$. Actually, any basis of R^3 is also OK, but I didn't give full credit for other bases without some valid explanation or work.

1c) A basis is $[1, 1, -1, 0]^T$ (essentially the same as the answer to 1a). Nullity = 1.

2) Combine the vectors into a matrix B and find its inverse. It has zeroes on the diagonal and $b_{12} = 1$ and $b_{21} = 1/2$. It is always a good idea to check your answer to any B^{-1} calculation, since it only takes a few seconds. I usually give less partial credit in this situation.

3) See text or HW.