MAS 3105 2003
Quiz 6 Key Prof. S. Hudson

1) Let S be the subspace of R^3 spanned by $\mathbf{x} = (2, 1, 1)^T$. Find a basis of $S^\perp$.

2) Let L be the derivative operator on P_3. So, $L(p(x)) = p'(x)$. Find the matrix representation of L with respect to the basis $[1, x, x^2]$.

3) Choose ONE of these.

a) Prove thm 5.1.1; that for nonzero vectors $\mathbf{x}, \mathbf{y} \in R^2$, $\mathbf{x^T y} = ||x||\ ||y||\ \cos(\theta)$.
b) If A and B are similar, then A^3 and B^3 are similar.
c) Derive the normal equations, used to solve least squares problems.

Answers:

1) Compute $N([2\ 1\ 1])$ using Ch.1 methods (set $x_3 = \alpha$, etc) and get $[-1/2\ 1\ 0]^T$ and $[-1/2\ 0\ 1]^T$. Since a basis is not unique, other answers are possible.

Instead of using Ch.1, some people guessed, but this didn't usually work out. Remember that dim S + dim $S^\perp = 3$, so the answer should be a list of *two* vectors.

2) Since $L : P_3 \to P_3$, and dim $P_3 = 3$, the matrix should be 3x3. Compute $L(1) = 0 = [0\ 0\ 0]^T$ which is column 1, etc. Get

$$A = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 2 \\ 0 & 0 & 0 \end{pmatrix}$$

3) These are in the text, except that 3b) was HW: Assume $A = S^{-1}BS$. Cube both sides and cancel the SS^{-1}'s and get $A^3 = S^{-1}B^3S$. So, A^3 is similar to B^3.