MAS 3105 — 3/29/04

Quiz 6 Key — Prof. S. Hudson

1) Let $\mathbf{x} = [1\ 2]^T$, $\mathbf{w_1} = [1\ 0]^T$ and $\mathbf{w_2} = [1\ 1]^T$. So, $F = \{\mathbf{w_1}, \mathbf{w_2}\}$ is a basis of R^2 and $W = [\mathbf{w_1}\ \mathbf{w_2}]$ is a nonsingular 2x2 matrix. Suppose $L : R^2 \to R^2$ is linear and

$$L(\mathbf{w_1}) = \mathbf{w_2} \quad \text{and} \quad L(\mathbf{w_2}) = \mathbf{w_1} + 2\mathbf{w_2}$$

a) Find the matrix representation of L with respect to F.

b) Find the coordinate vector $\mathbf{y}$ of $\mathbf{x}$ with respect to F.

c) Find the coordinate vector of $L(\mathbf{x})$ with respect to F.

2) Let $P_1 = (1, 1, 1)$, $P_2 = (2, 4, -1)$ and $P_3 = (0, -1, 5)$. Find a nonzero vector $\mathbf{N}$ that is orthogonal to $\overline{P_1P_2}$ and $\overline{P_1P_3}$

3) Choose ONE of these to prove (on the back).

a) State and prove the normal equations, used to solve least squares problems. You can start from a picture and can assume that $b - p \in R(A)^{\perp}$.

b) State and prove the Fundamental Subspace Theorem (5.2.1).

c) Prove that if A is similar to B, and B is similar to C, then A is similar to C.

Answers: The average was about 35/60, though the problems do not seem very hard.

1) This problem is based on Matlab exercise 1, page 220 (see also Example 3, page 202, etc).

$$A = \begin{pmatrix} 0 & 1 \\ 1 & 2 \end{pmatrix}, \quad \mathbf{y} = W^{-1}\begin{pmatrix} 1 \\ 2 \end{pmatrix} = \begin{pmatrix} -1 \\ 2 \end{pmatrix}, \quad L(\mathbf{x}) = A\mathbf{y} = \begin{pmatrix} 2 \\ 3 \end{pmatrix}$$

2) This is HW 5.2.5: $\overline{P_1P_2} = [1\ 3\ -2]^T$ and $\overline{P_1P_3} = [-1\ -2\ 4]^T$. Get $\mathbf{N} = [8\ -2\ 1]$ (any nonzero scalar multiple of this is also OK) by computing the nullspace of

$$A = \begin{pmatrix} 1 & 3 & -2 \\ -1 & -2 & 4 \end{pmatrix}$$

3) See text/lectures.