MAS 3105 June 16, 2005

Quiz 6 Prof. S. Hudson

1) Answer True or False (A is an m x n matrix, and S, T are subspaces of R^n):

If rank $A = n$, then $A^T A$ is nonsingular.

If $S \cap T = \{\mathbf{0}\}$ then $S \oplus T = R^n$.

$N(A^T) \oplus R(A) = R^m$.

If $\hat{x}$ is a least squares solution to Ax = b, then $A\hat{x}$ is the projection of b onto R(A).

Every norm on P_3 satisfies the triangle inequality.

2) Let $L : P_3 \to P_3$ be the transformation $L(p(x)) = xp'(x) + p(0)$. Find the matrix representation of L with respect to the basis $[1, x, x^2]$.

3) Choose ONE of these to prove.

a) Prove thm 5.1.1; that for nonzero vectors $\mathbf{x}, \mathbf{y} \in R^2$, $\mathbf{x^T y} = ||\mathbf{x}||\ ||\mathbf{y}||\ \cos(\theta)$.

b) State and prove the normal equations, used to solve least squares problems. You can start from a picture and can assume that $\mathbf{b} - \mathbf{p} \in R(A)^\perp$.

Answers:

1) TFTTT

2)

$$A = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 2 \end{pmatrix}$$

3) Both are in the book.