

1) [20pt] Let  $S$  be the subspace of  $R^3$  spanned by  $\mathbf{x} = (2, -1, 1)^T$ . Find a basis of  $S^\perp$ .

2) [20pt] Find the least squares solution to the system:

$$\begin{aligned}x_1 + x_2 &= 3 \\2x_1 - 3x_2 &= 1 \\0x_1 + 0x_2 &= 2\end{aligned}$$

3) [10pt] Compute the vector projection  $\mathbf{p}$  of  $\mathbf{x}$  onto  $\mathbf{y}$ , where  $\mathbf{x} = (2, 4, 3)^T$  and  $\mathbf{y} = (1, 1, 1)^T$

4) [10pt] Choose ONE of these.

a) In MHW2, problem 4.2, you created a 5x5 matrix,  $U = \text{hankel}(\text{ones}(5,1), 1:5)$ . Which MATLAB command were you asked to use on  $U$ , to quickly check that the columns are L.I.?

b) State the definition of *inner product*.

**Remarks and Answers:** The average was about 46/60. The scale for Quiz 6 is: A's = 52 to 60, B's = 46 to 51, etc (6 points per letter). I computed your semester average using your six quiz grades. The lowest grade was replaced by your current MHW average (on a 60 point scale), if that helped. The scale for these averages was the same as for Quiz 6. The results are in red, in the upper right corner of your quiz.

If I noticed an unexcused zero on one of your quizzes, or MHW, I made a note of it for you. If there is an error, or you forgot to bring in a doctor's note/etc, contact me ASAP!

Most Quiz 6 problems were relatively easy homework exercises.

1) Similar to 5.2.2a. Set  $A = (2, -1, 1)$ , so  $S = R(A^T)$  and  $S^\perp = N(A)$ , which is a Ch.1 problem. Normal GE methods lead to the basis  $\{(-1/2, 0, 1)^T, (1/2, 1, 0)^T\}$ . Other answers are possible, but you need to show your work for full credit. Trial and error is not reliable.

2) See 5.3.1a. The normal equations lead to  $[2, 1]^T$ . [You can use the formula in Thm 5.3.2 instead, since  $\text{rank } A = n = 2$ , but that's a little more work]. Notice that this vector solves equations 1) and 2) of the system exactly. Unusual - but so is the third equation!

3) See 5.1.3c.  $\mathbf{p} = [3, 3, 3]^T$

4a) rank

4b) See page 245.